7. CHESS AS A MULTI-STAGE GAME

Contents

Introductory remarks

Methods of formulating and solving a problem are many and diverse. Oftentimes, considerable ingenuity is required in order to determine which information belongs in the formulation of the problem and which in its solution.

I wanted to remind the reader of this well known phenomenon in order to emphasize the peculiar nature of the subject investigated in the present section of the book. Unlike general philosophical problems concerned with the fundamental questions of the 'essence of the world','the meaning of life', etc. which still remain open, chess represents a problem that has already been formulated in a very precise fashion: our task is to solve it. Although the problem of having to set local goals does arise in the course of advancing toward the global goal of the game, the latter is given to us beforehand.

Naturally, our analysis is greatly simplified when the problem is formulated in a complete and general form as is the case in chess. Clear cut rules and a certain amount of idealization allows us to concentrate on the inner structure of a system such as chess. Although chess is limited because of its rigid structure it is still very useful as a model in exploring certain problems in different areas of arts and sciences.

For many writers, the game of chess was an allegory of human relationships (for instance, Nabokov's novel Luzhin's Defense) or human psychology (like Poe's story The Murders in the Rue Morgue). Scholars and artists in many different fields though chess to be a good analogy to their craft. Direct associations between chess, especially its gambit openings and musical composition, its beginning in particular were noted by L. Rowell.

> He wrote: "One could easily write a history of music based on beginning mannerisms and gambits. In fact the analogy to the game of chess is a very good one: just as in chess there are certain combinations of opening moves (gambits) that implement the strategic principles of the opening game, namely (1) the rapid development of pieces and (2) control of the center of the board, in music there are various gestures which are designed to achieve some important strategic objectives of what we might call the "opening game of music."[1]

De Saussure compared the game of chess with language both in its formulation and methods of solution. For de Saussure the most valuable idea he extracted from chess was the precise understanding of the notion of the value of a piece; he emphasized the use of this idea in linguistics to help determine the significance of words in a sentence. De

[1]Rowell, L., "The Creation of Audible Time", The Study of Time IY, Papers from the Fourth Conference of the International Society for the Study of Time, Alpbach-Austria, ed. by J. Fraser, etc., New York: Springer-Verlag, pp. 198-210.

Saussure's observations on the connections between language and chess are so original that I want to quote them in full.[2]

Scholars did not employ chess merely as an analogy. Various methods of playing the game were used directly to solve some very practical problems and economic ones in particular[3]. The range of application of these methods is quite wide.

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[2]"In internal linguistics the picture differs completely. Just any arrangement will not do. Language is a system that has its own arrangement. Comparison with chess will bring out the point. In chess, what is external can be separated relatively easily from what is internal. The fact that the game passed from Persia to Europe is external; against that, everything having to do with its system and rules is internal. If I use ivory chessmen instead of wooden ones, the change has no effect on the system; but if I decrease or increase the number of chessmen, this change has a profound effect on the "grammar" of the game. One must always distinguish between what is internal and what is external. In each instance one can determine the nature of the phenomenon by applying this rule: everything that changes the system in any way is internal." (pp. 22-23).

"But of all comparisons that might be imagined, the most fruitful is the one that might be drawn between the functioning of language and a game of chess. In both instances we are confronted with a system of values and their observable modifications. A game of chess is like an artificial realization of what language offers in a natural form.

Let us examine the matter more carefully. First, a state of the set of chessman corresponds closely to a state of language. The respective value of the pieces depends on their position on the chessboard just as each linguistic term derives its value from its opposition to all the other terms.

"In the second place, the system is always momentary; it varies from one position to the next. It is also true that values depend above all else on an unchangeable convention, the set of rules that exist before a game begins and persists after each move. Rules that are agreed upon once and for all exist in language too; they are the constant principles of semiology.

Finally, to pass from one state of equilibrium to the next, or according to our terminology-from one synchrony to the next, only one chess piece has to be moved; there is no general rummage. here we have the counterpart of the diachronic phenomenon with all its peculiarities. In fact:

(a) In each play only one chess piece is moved; in the same way in language, changes affect only isolated elements.

(b) In spite of that, the move has a repercussion on the whole system; it is impossible for the player to foresee exactly the extent of the effect. Resulting changes of value will be, according to the circumstances, either nil, very serious, or of average importance. A certain move can revolutionize the whole game and even affect pieces that are not immediately involved. We have just seen that exactly the same holds for language.

(c) In chess, each move is absolutely distinct from the preceding and the subsequent equilibrium. The change effected belongs to neither state: only states matter.

In a game of chess any particular position has the unique characteristic of being freed from all antecedent positions; the route used in arriving there makes absolutely no difference; one who has followed the entire match has no advantage over the curious party who comes up at a critical moment to inspect the state of the game; to describe this arrangement, it is perfectly useless to recall what had just happened ten seconds previously. All this is equally applicable to language and sharpens the radical distinction between diachrony and synchronic. Speaking operates only on a language state, and the changes that intervene between states have no place in either state.

At only one point is the comparison weak: the chess player intends to bring about a shift and thereby to exert an action on the system, whereas language premeditates nothing. The pieces of language are shifted-or rather modified-spontaneously and fortuitously. The umlaut of Hande for hanti and Gaste for gasti (see p. 83) produced a new system for forming the plural but also gave rise to verbal forms like tragt from tragit, etc. In order to make the game of chess seem at every point like the functioning of language, we would have to imagine an unconscious or unintelligent player. This sole difference, however, makes the comparison even more instructive by showing the absolute necessity of making a distinction between the two classes of phenomena in linguistics. For if diachronic facts cannot be reduced to the synchronic system which they condition when the change is intentional, all the more will they resist when they set a blind force against the organization of a system of signs." (pp. 88-89).

"Finally, not every idea touched upon in this chapter differs basically from what we have elsewhere called values. A new comparison with the set of chessmen will bring out this point (see pp. 88 ff.). Take a knight, for instance. By itself is it an element in the game? Certainly not, for by its material make-up--outside its square and the other conditions of the game--it means nothing to the player; it becomes a real, concrete element only when endowed with value and wedded to it. Suppose that the piece happens to be destroyed or lost during a game. Can it be replaced by an equivalent piece? Certainly. Not only another knight can be declared identical provided the same value is attributed to it. We see then that in semiological systems like language, where elements hold each other in equilibrium in accordance with fixed rules, the notion of identity blends with that of value and vice versa." Saussure, F., Course in General Linguistics, New York: McGraw Hill Book Co., 1959, p.110.

[3]Botvinnik, M., "Decision Making and Computer", Advances in Computer Chess, 3, ed. by M. R. B. Clarke. Oxford: Pergamon Press, 1982, pp. 169-179; Nigan, R., "Operations Research/Management Science and Chess", RCA Engineer, Vol. 29, No. 4, 1984, pp. 69-73.

For instance, in 1984 the well known Boston Consulting Group published a brochure entitled "Business Chess". It notes that despite chess' relative simplicity it can still serve as a model to help solve economic problems. This point is substantiated by the following observation:

> "...they [chess and economics] are systems of competition, and the competition involves indirect consequences."

Authors of the brochure provide several examples of chess being utilized in the world of business. These examples deal with various chess concepts such as gambits, establishing the value of the pieces under different circumstances, recognition of a position, various chess techniques elaborated over the course of time, and the need to learn. Jumping ahead a little I want to note that the authors of the brochure omit the use of prior experience in formulating and solving local problems, i.e. problems arising within the search tree.

Chess ideas were put to much wider use by the former world chess champion M. Botvinnik. In particular, Botvinnik's article in the newspaper "Pravda", March 30, 1981 described a program for scheduling the repair work of the Central Dispatching Office of the United Energy System of the USSR. This program was based on chess algorithm called "Pioneer 2" which he helped develop.

Naturally, chess methods found their immediate application in the field of artificial intelligence. H. Simon stressed the role of chess viewing it as a

> "...microcosm that mirrors interesting properties of decision-making situations in the real world."[4]

I only want to note that many scholars consider chess a credible source for studying artificial intelligence because the game, due to its clear cut and rigid rules, yields to formal representation. Chess possesses a symbolic language which can be used to define the game. Note that this language can be understood by people with different native tongues. In recent times this language not only acquired a chess notation which refers to individual pieces, their coordinates and moves, but it also incorporates the analysis of how the game is actually proceeding.[5] The symbol + means advantage for white; Δ - with an idea; ˙ - the only move, etc. Positional parameters are also given a symbolic representation. ∞ denotes connected pawns, ◦...◦ stands for isolated pawns, etc.

But the major advantage of chess in the field of artificial intelligence. It seems to me, at least the following three features make chess so eligible in this area.

The first is due to the fact that chess represents a model of system's development which does not yield (perhaps even theoretically) to a constructive algorithm which would link completely (in a sense of complete certainty of the outcome) a given step with the final outcome.

The other peculiar aspect of chess is that it can be viewed as a multi-stage game. A stage, from an operational point of view, represents a certain interval of the overall process which employes a particular method of operation. Thus, multi-stage processes give rise to the problem of finding an adequate set of methods for each stage and their subsequent integration. Chess is a treasure chest of experience when it comes to multi-stage processes, a fact which is evident in the advanced theory of openings, middle game, and endgame.

The third feature pertains to the positional method. Of all the various methods used in this ancient game the positional method practically owes its advancement to the twentieth century.This very original method incorporates unique ideas regarding analytical

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[4]Simon, H., "Theories of Bounded Rationality", Models of Bounded Rationality, vol. 2. Cambridge: The MIT Press, 1982, p. 412.

[5]"Chess Informant", 1980.

evaluation of a position - evaluation which can be reduced to numbers. I know of no other area exhibiting the same precision when it comes to evaluating a state of the system. Axiology has not as of yet absorbed these revolutionary ideas into its arsenal. Nevertheless the role of positional method is enormous for it forms the foundation of the aesthetic method of judgement by means of which indeterministic processes permeating all different areas of human endeavor are implemented.

Finally, chess is highly useful in the field of artificial intelligence in that it yields to an experimental verification of various ideas. A famous Soviet mathematician A. Kronrod, one of the pioneers of computer chess in the Soviet Union, once wrote that the role of chess in artificial intelligence could be compared to that of a drosophila fly in genetics.[6] On the one hand, chess' peculiarity as a multi-stage game, its relative simplicity as compared to many other systems, and on the other, fast attainment of the final state[7] and statistics that can be gathered because we can always come back to the initial state, give us reasons to believe that chess can become a very useful experimental tool.

Now I want to say a few words about the structure of the present section.

It has already been said that the end goal in chess is given. Still a few comments regarding this aspect of the problem are in order and this constitutes the subject matter of the first section. It basically consists of comments regarding the problem of goal formulation. This approach is different from a more analytic one used in discussing the solution of the problem. The greater part of the chess section of the book deals with the latter topic. The section is divided into four chapters.

The division was based on the generality of the problem in question. This criterion gave rise to the following grouping: global problems pertaining to the overall flow of the game, local problems embracing a few moves, and problems occurring at intermediate levels - stages of the game.[8] Each of these subjects comprises a chapter. Importance of the problem of evaluating any given state of the game warranted a separate chapter.

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[6]Kronrod, A. S., "Machine is Getting More Intelligent", Pravda, March 15, 1967.

[7]Constraints on the duration of the game is not without drawbacks. For instance, each side in a given game is represented by a single player. Real life development of an idea may require significantly more time than the life span of its inventor. Here a serious problem of ensuring proper transfer of power comes under consideration. Chess could perhaps mimic this aspect if something like a relay race was introduced with the game being handed from one player to another. Here participant selection would become an issue.

[8]Division of the field into a macro and a micro, as is the case with economics, seems inadequate for our discussion because certain intermediate stages significant in themselves are left unaccounted. Our case is closer to the classification of stages in planning where a distinction is made between strategic, mid-term, and tactical planning.

Chapter 1. The set up of the chess game

§1. The place of chess.

Let us start by immersing chess in a more general system.

By its nature chess belongs to the realm of art rather than science (taking the criterion of art to be the creation of models which do not necessarily reflect reality). What distinguishes these models from the scientific ones is that they do not require experimental verification to determine whether or not they correspond to some real phenomena outside of themselves.[9]

But if we view science as that sphere of human thought which produces objective knowledge, i.e. knowledge that can be passed on to other people who can use it with the same results, then chess should be part of sciences since methods used in the game allow for this kind of knowledge to be formed.

Therefore, chess occupies a very peculiar place in the system. It can be included among the arts as well as sciences. Moreover, although the formulation of the game places it in the category of art chess does reflect some very important aspects of reality. Judging by the methods used in the game chess can be classified as a science although an aesthetic method which prevails in the creation and perception of art plays an important role in chess.

So what is the role of chess as far as it serves these two dieties? Just like other games chess fulfills many functions: it is played for recreation, it is used to teach certain techniques of decision making, and to influence the behavior of people. All this is accomplished by replacing the "real" actions with their model simulation. Chess is rather peculiar in this respect. I shall explain what I mean by this statement later on.

Like many other games chess belongs to the class of games whose purpose is to simulate certain aspects of real life. It may seem at first glance that chess merely reflects real life conflicts, or to be more precise, it simulates military combat.

But this approach to chess and its connection with reality is rather limited and superficial. Thinking of chess as a constructive model capable as such (i.e. its specific methods of play ignored) of affecting interrelationships among people we begin to see that

[9] This is precisely the criterion used in awarding the Nobel Prize. It is not an accident that the peace prize and the award for literature are placed in a different category. Nobel prize is not awarded for mathematics since according to this criterion mathematics belongs in the realm of art. (Bitter tongues say that Nobel excluded mathematics from the list of disciplines for which the prize is awarded because his wife's lover was a mathematician).

its connection with reality is much broader and deeper.

It seems to me that reducing chess to a game whose sole purpose is to simulate a conflict situation is an oversimplification. To clarify this point I want to call to the attention of the reader the following classification of relations, including conflicts, proposed by J. Gharajedaghi and based on the ideas of R.Ackoff.[10]

Table 1. Types of interrelations

Means	Ends	
	Incompatible	Comp
Compatible	Collaboration	Cooperation
Incompatible	Conflict	Competition

This taxonomy gives rise to four types of relations. Two of them correspond to conflict situations. What makes competition different from a conflict is that the goals of the participants are compatible while the means are incompatible. Here, the term goal stands for a noble goal such as the enjoyment of the game; the goal of winning merely represents means of achieving this end.

This idea was expressed independently and in different words by A. Koestler. He wrote:

> "Edward Lasker (namesake of the great Emanuel, and himself a grand master) wrote a revealing book with the title: Chess for Fun and Chess for Blood. But "fun" is the wrong word; what he meant was that the game of chess is the perfect paradigm for both the glory and the bloodiness of the human mind. On the one hand, an exercise in pure imagination happily married to logic, staged as a ballet of symbolic figures on a mosaic of sixty-four squares; on the other hand a gladiatorial contest. This dichotomy is perhaps the main secret of the game's astonishingly long history."[11]

Actually, there are some forms of chess where players relate to each other in ways other than purely competitive. I mean the 4-player version of chess which was played in India. There were two sets of pieces - green and white on one side and black and red on another. These two groups related to each other in a competitive mode. Within each of these groups relations were of a cooperative type since both the ends and the means of

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[10]Gjarajedaghi,J., Toward a Systems Theory of Organization. Seaside: Intersystems Publications, 1985, pp. 41-42.
[11]Koestler, A., The Hill of Achilles. London: Hutchinson of London, 1974, p.196.

the participants were compatible.

So according to the classification proposed above chess represents a game which simulates competition since the goals of the participants are compatible while the means are not.

§2. The function and the structure of the game.

Now, let us take a look at the final product of the game and its role in the fulfillment of the aforementioned functions.

Chess belongs to the class of games having a well defined final goal (unlike other games lacking such goals - for instance, a man running around playing with a ball). The goal in chess is to capture one very special material object - the opponent's king. Victory is characterized by the state of one single piece, the king. The state of the system as a whole is of no consequence at all. In other words, the price of a victory is irrelevant: you can lose almost all your pieces as long as you capture the enemy's king. So Pyrrhic victory which is so very dangerous for systems having no end goals (they develop continuously) is impossible in chess. Actually Pyrrhic victory has its significance for chess but only as far as intermediate stages of the game are concerned.

We can imagine a form of chess having a different objective with an overall state of the system being evaluated at the end of the game. Here, it becomes necessary to assign external values not only to the king but to other pieces as well. The more pieces are left on the board, both your own and your opponent's, the greater is your victory.

Now, on the structure of chess. There is a considerable variety of pieces in chess. They differ in the type of moves they are allowed to make.

This feature distinguishes chess from many other games where the number of objects manipulatable by the players is limited both in quantity (soccer, baseball) and quality (bowling, checkers). The physical weight of the piece and their volume are insignificant. Pieces operate within the limited space of the board having a rigid geometric shape. Especially noteworthy is the fact that each piece occupies and is allowed to occupy the same area of the board as any other piece. This indicates a certain degree of equality among the pieces.[12] I do not know whether this rule was introduced because of some special considerations rooted in the social structure or just to simplify the game.

Equality in chess is not limited to certain properties of the pieces. The basic structure of the game presupposes that both players are equal both in the quantity and the quality of the pieces they have as well their starting positions.

The fact that the initial conditions for both players are the same is evident in the present day version of the game - the shape of the pieces is the same for both players but the color which has no special symbolic meaning is different.[13] At the same time there exist some forms of chess where this feature is modified while all other rules are basically the

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[12]A desire to create equality among members of society arises not only from altruistic considerations but also from the fact that society with inequality seems harder to integrate.

[13]A book by Mackett-Beeson, A.E.J., Chessmen. New York: G.P.Putnams Sons, 1968 shows photographs (some of them in color) of chess sets used in different countries. These photographs are accompanied by a brief note on the history of some of the forms of chess, the materials from which the pieces were made, and so on.

same as in regular chess. Mongolian chess is interesting in this respect: it mimics the universal concepts of good and evil.[14] In keeping with their tradition the Mongols considered red (as well as yellow) to be masculine or spiritual color - the color of the sun; green was seen as feminine or earthy color - the color of the Earth. Consequently the base of the pieces belonging to the two players is colored differently: they are either all red or all green. (The upper parts of the pieces are painted with both colors perhaps to reduce the stress on the players. It may also signify that each player possesses characteristics associated with both colors). For a long time the Mongols considered Chinese to be their arch enemy. Their attitude is reflected in the shapes of the pieces. The expressions on the faces of the "kings" representing the Mongolian prince and the Chinese ruler are quite different. The queens differ in shape as well: one is in the form of a lama - a Buddhist lion and the other is in the form of a tiger-devil borrowed from the primitive pre-lamaic religion. The shapes of the camels, horses, and carriages are the same for the two players. But the pawns are very different: they are carved into shapes of different birds meant to express the conflict between the spiritual and the material, between the heavenly and the earthy.

Although the initial position and the number of pieces is the same for both players the game does reveal differences in their innate abilities and knowledge. Equality in the starting position and number of pieces which the two players have at their disposal vanishes as the game progresses and eventually leads to a victory (defeat) for one of the sides. Equal starting conditions are reinstated at the start of each new game.

Digression 1.

The concepts of equality and inequality in chess stimulated me to apply these general ideas to social systems. The problem of social inequality is rooted primarily in the fact that initial material resources are not distributed equally among the members of society. All other conditions being equal people who start out with more material wealth can achieve more in life than people who started out with less. Here inequality in wealth should not be confused with inequality in individual abilities - inequality in drive, energy, ability to perform various tasks, and intelligence - a quality which integrates all these characteristics among themselves and the environment. The relative impact of innate abilities, i.e. the genetic code and the time spent in the womb as opposed to the role of the environment is a very complicated issue. This age-old debate is outside the scope of the present work. Personally, I think that innate abilities are crucial in the formation of an individual and that they should be taken into account in our attempts to resolve social problems.

Finally, inequality in material wealth and individual abilities should not be confused with the fruits of one's labor which results in inequality in current income. This kind of inequality may be caused by the two-fold inequality mentioned above, i.e. inequality in the initial conditions and in innate abilities.

In his wonderful work called "white revolution", V. Z. Jabotinsky[15] examined all these aspects of social inequality from a very interesting perspective. The "socialist" approach to the problem of inequality is limited because it tries to make both the initial conditions and

[14]Camman, S., "Chess with Mongolian Lamas", Natural History, November 1946, pp. 407-411.
[15]Jabotinsky, Z., Selected Works. Jerusalem: Aliia library, 1978.

the current income of all individuals equal once and for all. The drawbacks of the "bourgeois" method is that it tends to perpetuate these inequalities.

Jabotinsky thinks that the most effective solution is to combine the socialist principle of equality in the starting conditions with the bourgeois principle of inequality in current income. He noted that equality of initial material resources is important to ensure a favorable start for every individual. At the same time he emphasized the need for inequality in the distribution of wealth in order to stimulate people to develop their innate abilities. How can we combine these two principles? Jabotinsky showed that certain laws on the redistribution of wealth will help society move in the direction of equality in the initial conditions without resorting to bloody revolutions; here the spirit of achievement will be fueled by the inequality in incomes.

To illustrate his idea Jabotinsky borrows an example from the Old Testament. There is a law in the Old Testament which states that the land purchased at some point in time must be returned to its owner after a period of 50 years. This is where the Hebrew word "jubilee" comes from. It found its way into the Roman lexicon and is now part of many European languages.

This kind of a "white revolution" is different from the "red revolution." The latter involves violent means used in order to change the initial conditions of the people.

Jabotinsky notes that the laws of the Old Testament were appropriate only under the then prevalent historical conditions but the principle used is still of interest. It can be applied to new situations by enacting appropriate laws which would ensure a peaceful solution to the problems arising in the course of development.

§3. The rules and the operator of the game.

Now, let us come back to chess. I want to talk about the process aspect of chess. As far as the set up of the game is concerned it manifests itself in the rules of the game. Rules enable players to preserve the unity of their noble goal: to enjoy the game while keeping the spirit of competition (in terms of the means) alive. These rules which are rather extensive pertain primarily to the moral requirements placed upon the players. Oftentimes they are not taken into account because of the emphasis on the rigidity of the rules of the game itself.[16]

The use of chess as an allegory of moral behavior has a long history. A book devoted exclusively to this topic appeared as early as14-th century. It was written in Latin by **Jacobus de Cellosis** and was called **Libellius de Moribus Hominum et Officiis**

[16]In his book on Stalin Trotsky gives the following account of the moral relationship between the player and the game:
"The laws of political mechanics once formulated by Machiavelli were for a long time considered extremely cynical. Machiavelli viewed the struggle for a power as a chess theorem. For him, moral dilemmas do not exist just as they do not exist for a chess player or for an accountant whose task is to do the most expedient thing under the circumstances." Trotsky, L., My Life, 1985, p. 14.
I disagree with Trotsky when he says that chess players face no moral dilemmas: the morality of a chess player should not be reduced to the way he moves the pieces. Psychology plays an important role in one's play and sometimes one oversteps moral boundaries. If we compare the behavior of B. Spassky towards B. Fisher in their world title match in Iceland with the behavior of A. Karpov towards V. Korchnoi and G. Kasparov in their title matches, we see the impact of one's moral stature on his conduct. Morality is also important in the course of the game itself: for instance, whether one player "terrorizes" another with his rude behavior which may be legal but is morally unclean thus ruining the aesthetic side of the game.

Nobilium. It was translated into all the European languages 50 years after its first publication.[17]

So, what are the rules of chess that are so important from the moral point of view?

First of all, the game is governed by a set of strict rules which both players must agree to obey. In chess battles the role of referee-judge is rather minute compared to such sports as football, basketball, boxing, etc. The gap between chess and real life is evident form the fact that in many cases there are no rules guiding the behavior of the conflicting parties. Even if such rules do exist there may be no referee to spot the violations.[18]

Rules of chess presuppose that the outcome of the game depends solely on the strength of the players. His majesty chance plays no explicit (or as such) role in the game. This makes chess different from many other games - card games, dominoes, lotto, roulette, dice, backgammon, etc. The role of chance in these games is big enough to determine the outcome of at least one round. These games require a large number of repetitions in order to eliminate the the luck factor.[19] To a certain extent these considerations also apply to chess as we know it. White pieces received by chance give the player certain first move advantage in that particular game. The "first move" advantage is largely neutralized when players play many games with each other.

On occasion the first move privilege does become significant even in a long match between the same players. World championship matches have demonstrated this a valid concern in cases where the last game turned out to be decisive.

Jumping ahead I would like to note that the importance of the first move stems in part from the fact that the starting position has many pieces constricted in movement. First move advantage could be neutralized by introducing a rule called a "tabia". This rule would have the game start at a particular position judged to be equal by both players.

Present-day rules of chess which eliminate the role of chance in the course of the game itself were not always the same. At one time a die was tossed before a player made a move. It seems that during the middle ages when Europe made an attempt to eradicate gambling the role of chance as such disappeared from the game.

We should mention another important provision having to do with the chance factor: at each moment of time each side knows the position of the opponent's pieces and therefore every move made by the opponent up to that point. This kind of information can be helpful in predicting the future moves of the opponent.

One form of chess called Kriegspiel lacks this rule. Kriegspiel is the same as regular chess except neither player knows the moves made by his opponent. Therefore, each player must be told by a referee who keeps track of the game whether or not his moves are legal.

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[17] Michigan University Library has a microfilm of the work by Fuller, C., A Critical Edition of Le Jeu Des Eschs, Moralize. Ann Arbor: University Microfilms International, 1976, which has a detailed analysis of the French translation of this book.

[18] Huizinga,J., Homo Ludens. New York: Roy, 1950.

[19] So it is not an accident that international competitions only allow games where the role of chance (when it can favor one participant more than another) as "an additional player" is reduced to a minimum. Poker tournaments are permitted since each round takes a short time to complete so a lot of rounds can be played within a reasonable period of time. Statistically this helps eliminate the luck factor. Bridge competitions are organized in a very "tricky" way: each side has two teams which play simultaneously: in each round, one team plays with its own cards and the other has the cards of the first team's opponents.

Absence of chance per se and complete knowledge of the opponent's position idealize chess as compared with real world conditions where these factors may play a significant role.

Let us now examine some rules of chess as far as they reflect society's moral code. First of all, there is a rule which states that a player is not allowed to remove his own pieces from the board, i.e. to "kill" the piece "himself." One can offer a piece as a sacrifice leaving it up to the opponent to decide whether to accept or reject the offer. This rule is very strict: a player is not allowed to capture his own piece even when its removal from the board would prevent an immediate defeat or lead to an immediate victory. This rule is somewhat reminiscent of the moral principle which states that those in power cannot kill their own countrymen under any circumstances. At the same time homicide is permitted in military combat - both those killed by the enemy directly and those offered to the enemy for the ruler's own benefit. This moral principle is quite realistic. It is fully applicable to such countries as Sweden, Denmark, Switzerland, and others. These countries not only forbid the murder of innocent people for whatever reasons but they have also done away with capital punishment for criminals; at the same time these countries have armies and are ready to kill their enemy and sacrifice their own population.

Another rule of chess states forbids the use of any piece captured from the opponent: the piece leaves the board once and for all. This rule reflects a belief that each participant in the combat should remain loyal to his country and even if captured he should not be fighting against his own army on the side of the enemy. But a Japanese version of chess called Shogi[20] permits the use of the captured piece as your own (it is somewhat inferior because it is not allowed "to advance in rank" immediately; more on this below). The captured piece can be placed on any unoccupied square on the board.[21] Perhaps this rule reflects the conditions prevalent in the wars which took place between the Japanese feudals during the period when Japan was a less centralized country.

Stalemate rule is rather curious. The winning side must conform to another constraint of avoiding immobility of the opponent's king if all his other pieces are also immobile. The roots of this rule are unknown to me. Perhaps it arose in the middle ages if we interpret the rule as preventing situations where an attacked knight has no place to move before being directly struck and lacks mobile troops to continue the struggle. In other words "Do not hit a man when he is down".

Finally I want to note that the rules of chess restrict the capacity of a piece to change its "social status," i.e. its role in the game. The attributes of a given piece are fixed once and for all. It may gain or lose in strength depending on its position on the board or the positions of other pieces. (For instance, a bishop can only make diagonal moves. On an empty board, it can make anywhere from 7 moves if placed on the edge of the board to 13 moves if placed in the center of the board).

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[20] Games of the World, ed. by F. Grunfeld. New York: Ballantine Books, 1975, pp. 77-83.

[21] This rule as well as the "advancement in rank" rule are easy to implement in Shogi since all the pieces have the shape of a pentagon. Pieces are distinguished by the hieroglyph drawn on top of them. The direction in which the pentagon shaped piece points determines its "owner."

A pawn represents an exception to this rule. It can "be promoted." In fact once it reaches the opposite edge of the board it must be promoted to any other piece including a queen but not a king.

We should again note that Shogi provides far greater opportunities for the pieces to change their "social status". Pieces can be promoted once they reach one of the rows near the edge of the board. They also do not have to change immediately upon reaching a certain row: a player can change them any time after they have acquired the right to "advance."

Everything said so far is related to the social and moral attitudes toward an individual. What is the role of innate abilities in determining a person's life? Does social mobility depend on personal success?

Let us now examine the notion of an executor-operator of the chess game: is this a man, a machine, or some combination of the two? Chess presents us with all of these combinations: man against man, man against machine, machine against machine. Here, the term machine stands for any artificial device which plays the game. History of chess is abundant with examples of these kinds of machines turning out to be "imposters" - in one case a man was put inside a machine but it looked like the moves were made by a mechanical device.

So, what are the requirements placed upon the operator of the game considering that each game must be played in real time, i.e. only a finite amount of time is allotted to each game?

Jumping ahead a little I want to note that a machine algorithm for playing chess does not have to imitate that of a man if only because a machine is capable of calculating variations much faster than a human being, it does not get "tired," it can house a huge memory, and so on. At the same time a number of very difficult problems attributable to the role of individual experience arise when we try to design a machine algorithm of the game. There are examples of technological devices whose principles of operation differ from the analogous objects occurring in nature.[22]

The other approach assumes that computer chess programs must be based entirely on the principles of the game elaborated by man. Much can be said in support of this approach. For instance, there is an entire area of science called bionics which deals with design of new technologies based on the principles found in the living systems (the principle used by the bats to navigate was used in the development of radar); late 80-ies witnessed the development of the idea of designing computers capable of pattern recognition which would mimic the neuronetwork of the brain as opposed to fast and powerful sequential computers.

It is hard to say at this point which approach or which of their combinations is most beneficial to the development of a chess algorithm. In any case experience with computer chess has revealed the need to combine the ideas of incorporating both the human experience and the special capabilities of a machine.[23]

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[22] Stevenson's genius consists of having discovered a new principle for transforming the back and forth motion of the steam engine into the rotary motion of the wheel. An attempt to build a steam locomotive in the image of a horse - steam engine being its heart, ended in a failure.

[23] Adelson-Velskii, G., et all, Machine Plays the Game, Moscow: Nauka, 1983, pp. 60-71.

At the present time, research of chess algorithms is linked primarily with computers; an elaboration of such an algorithm for the human players is not considered to be a crucial issue.

Important results have been achieved in the field of computer chess. The best chess programs of today play almost as well as grandmasters: on September 25, 1988 chess program Hitech won a match with grandmaster A. Denker by a score of 3 1/2 to 1/2.[24]

Sceptics who did not believe in the future of computer chess such as the philosopher Herbert Dreyfus from Berkley who claimed that a computer will never beat even a ten year old have been put to shame.[25] But overly optimistic predictions have not been fulfilled either. For instance, in 1957 Professor H. Simon from Carnegie-Melon University made a statement that in 10 years computers will play better than the world champion. But, alas!

Still, I tend to side with the optimists. Although their expectations may have been too high they stimulated the development of computer chess and achieved a number of great victories on the way. Therefore I feel supportive of the statement made in 1986 by Hans Berliner from Carnegie-Melon that by the year 1990 his chess program has a 50% chance of beating a world champion, both among people and machines. A pledge of his success lies in the fact that he created the first computer program to beat the world champion in a board game - his backgammon program. Also, by mid 1986 his chess program Hitech was considered the ninth best player in Pennsylvania and 200-th best player in the United States and it still has a lot of room for improvement.

§4. The genesis of the game.

Now let us move on to the historical aspect of the game. Chess is one of the most ancient games. Although traces of games similar to chess have been found on ancient Egyptian sculpture[26] most scholars agree that the present day version of chess (or similar version to be precise) first appeared in India in 7-th century A. D.[27]

Some attribute the invention of chess in India to one of the game's aforementioned functions, namely it simulates military combat. It is held that the Buddhists who were quite influential in India at that time were able to introduce the game to the people. It was in harmony with their ideology which renounced violence and saw chess as a substitute for war.[28] There were even instances when two European countries which were at war with each other settled their disputes at the chess board. Unfortunately the reverse also took place. A player, seeing inevitability of unfavorable outcome would take some drastic measures against his opponent, even murder.[29] As a matter of fact during the middle ages chess pieces could have been used directly as a weapon. Considering their weight and the

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[24]Schonberg, A., "For First Time, a Chess Computer Outwits a Grandmaster in Tournament", The New York Times, Sept. 26, 1988.
[25]See Time, Oct. 28, 1985, p. 88.
[26]Lasker, E., Lasker's Chess Manual, Moscow: OGIZ, Fizkultura i Turizm, 1937, p. 5.
[27]Games of the World, ed. by F. Grunfeld. New York: Ballantine Books, 1975, p.63.
[28]Koestler,A., The Hill of Achilles. London: Hutchinson of London, 1974, p.198.
[29]Fuller, C., A Critical Edition of Le Jeu Des Eschs, Moralize. Ann Arbor: University Microfilms International, 1976, p.15.

size of the board a "mutual exchange" of material could lead to some very unsettling results.[30]

The present day form of chess is special in its variety of material objects - pieces, their ability to move around, and rules on how they ought to interrelate. This distinguishes chess from many other games that are much more limited in their potential. At the same time, chess in its present-day form is a rather conservative game compared with other systems (especially social ones) in which new objects and new rules continue to appear. Not only is the variety of pieces limited to what it is now but the current rules of the game prevent new pieces or new rules from being introduced. If such changes do take place they are very rare and usually lead to the creation of a new version of the game.

History of chess is rather rich in this respect since the game has undergone some profound changes in the course of its evolution. Here we should note that there are several major forms of chess now in existence. Beside the game properly known as chess there is Chinese chess and Japanese chess known as Shogi.

At the same time, chess players are trying to invent new original forms of the game.[31] Along with modifying the rules of the game or inventing new kinds of pieces with unusual abilities new shapes for the board have also been proposed. The board can simply be increased or decreased in size (maxichess and minichess), its shape can be changed keeping it two- dimensional (like hexagonal chess), or it can be constructed in three dimensions (cylindrical, spherical, or torus chess); theoretically, we can even think of multi-dimensional chess. These new forms of chess were not merely a fruit of idle imagination. Many of these new forms of the game (with different boards, new rules and new pieces) were actually implemented and some were even played at a competitive level.[32]

In summarizing the above discussion concerning the formulation of the problem (the role of chess in the solution of the problem shall be discussed below) chess represents a model which not only simulates reality, especially its social aspects, but also has a big constructive impact on the development of the world.

Chess is an example of competition which is subject to a rather elaborate set of rules: the board, the pieces, and the rules of their "conduct." Chess also represents an interesting model of a social system where, on the one hand, the initial conditions of the two players are the same and they can be reconstructed at the start of each new game and on the other, the outcome of the game depends solely on the individual abilities of the participants.

Nevertheless chess presents a rather limited view of the world because it has less variety that real systems: "participants" (pieces) and rules for manipulating them are given, i.e. neither the pieces nor the rules can be changed within the system (not to be confused with new possibilities introduced from the outside as novel versions of the game). Also the game possesses a final and a rather limited goal (to capture the king without any concern for the other pieces); the rules are reduced to a form of competition where chance plays no explicit role and the positions of all the pieces are known to each of the participants at each

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[30]Koestler,A., The Hill of Achilles. London: Hutchinson of London, 1974, p.199.

[31]Gik, E. J., Chess and Mathematics, Moscow: Nauka, 1983. Examples of modifications given here are taken from the two chapters of the book - "Games played on unusual boards" and "Magical chess".

[32]Capablanca who invented maxichess played a match with a Hungarian grandmaster Marotzy. In 1929, the first tournament in hexagonal chess was held in Europe.

moment of time. A ban on removing one's own piece (taking them off the board) under any circumstances, pieces being limited in their ability to change their "social status," and so on are all rules which make chess further removed from reality.

Discussion which follows focuses on the solution of the problem of chess. We shall examine a number of general ideas which come up despite or rather as a result of the simplicity of the game.

Chapter 2. The game at the global level.

§1. Optimization vs. chance. An inexact problem.

So the game of chess as a problem has been formulated. Its definition is exhaustive: the goal, the initial conditions and the rules governing the operations allowed in the course of the game.

We are presently faced with a task of designing and implementing a method or an algorithm for solving this problem. Design of a chess algorithm requires a clear understanding of the nature of the operator or the executor of this algorithm since this algorithm must be executed in real time. Our subsequent discussion is based primarily on the recent developments in computer chess and, where pertinent, the experience accumulated by the chess players over the course of many years.

Design of a chess algorithm involves furnishing a given "player" with an optimal "trajectory" of development, i.e. a sequence of moves which maximizes "his" objective function. Formally this algorithm must maximize a function whose arguments can assume the values 1, 1/2, or 0 - a win, a draw, and a loss respectively. Theoretically a search for an optimal strategy for one player must be accompanied by the same kind of search for the other player since it is assumed that the opponent is also rational and plays the "best" possible moves. Finding an optimal strategy in chess which is a zero sum game means designing a max-min algorithm where a maximum gain for one player is equal to a minimum loss for the other.

Now, before attempting to implement this kind of an algorithm let us ask ourselves the following question based on one of the principles of "non-constructive mathematics": "Does an algorithm of the game exist at all?"

Theoretically, such an algorithm does exist since the number of mating positions attainable during the course of the game is finite. True, the number of such positions is huge. Even if we assume that the game only lasts 40 moves (5-6 thousand moves represents the limit if positions are not allowed to repeat) there are 10^{120} possible variations from the starting position. It would take a computer 10^{90} years to go through all these variations even if it spends one micro-micro second on each one.[33] So although an exhaustive search would require only a finite amount of time it is irrelevant to the human time scale. Therefore a real solution to the problem calls for an ordered search with the number of operations bounded by some reasonable number. Ideally an ordered search ensures an optimal outcome of the game. This certainty stems from the optimization principle which consists of finding an optimum point by means of a completely ordered procedure, i.e. a procedure proved to be convergent and which searches through all the variations eventually eliminating all the inferior ones. In the last few decades research in the areas of linear (including integer solutions) and dynamic programming has proved the

[33] Shannon,G., 1950, "Programming a Computer for Playing Chess", The Philosophical Magazine, vol.XLI, 1950, pp. 259-260.

existence of algorithms which lead to an optimal solution in a finite number of steps. There are rigorous proofs of the fact that these procedures converge to an optimum point.[34]

Chess, it turns out, is too complex for present day optimization algorithms. Unfortunately chess literature that I am familiar with does not discuss the specific reasons why chess does not yield (in principle) to these optimization algorithms.

We should add that an effective algorithm of the game cannot be reduced to elaborating an optimization algorithm. A useful tool in designing an effective algorithm based on selective search procedures takes into account the specific structure of the board and the rules of the game. K. Thompson, for instance, designed an algorithm for 5 piece (including kings) endings which generates an exact move corresponding to the best possible course of action.[35] These retrograde algorithms (i.e. they start from the end - from winning positions) are basically based upon an exhaustive search and so do not, strictly speaking, belong to the class of optimization procedures. Nevertheless, the search is greatly reduced by taking into account the symmetry of the board. Under certain conditions a solution corresponding to a particular square can be extended to three other squares.

We should note in passing that algorithm efficiency can be further improved through the use of specialized chess computers.

My subsequent analysis deals only with those chess algorithms which employ all kinds of methods of truncating the search tree and which do not rely directly on board structure, rules of the game, or the speed of the computer.

Since an optimization-like search of all the variations is denied making <u>random</u> moves may seem like the only other way to play. Indeed we rarely know all the consequences of a given move so we should allow his majesty chance to decide the move for us. Since the number of moves is finite and present day rules limit the game to approximately 6000 moves before it exhausts itself completely, random move generation would lead to an end of the game within a reasonable period of time. But this method deprives the game of all its meaning. The game loses its cohesive structure.[36]

I do not mean to play down the role of chance in chess completely. Research of algorithms of mathematical programming (with proved convergency) has shown that under certain conditions we can achieve good results by combining ordered and rigid procedures for determining the best course of action at the start of the iterations (such as the simplex method) with a random method (like the Monte-Carlo method) closer to the end of the program.

We thus face the problem of designing an approximate algorithm of the game hoping that it would produce good results within a reasonable period of time. Unlike optimization or exact algorithms, these algorithms could be called good-non-optimal or approximate-inexact

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[34] It can be proved that algorithms in linear programming must perform an exhaustive search in order to find an optimal solution to a certain set of problems. In practice such a search is not required for an optimal solution.

[35] Thompson, K. "Retrograde Analysis of Certain Endgames", <u>ICCA Journal</u>, Vol. 9, No. 3, 1986, pp. 131-139.

[36] To get pleasure from the <u>game itself</u>, the search must be ordered in some creative fashion. The fact that computers receive no such pleasure is a result of them lacking a program of creation, i.e. they do not form these programs of creation by themselves. It seems to me that an emotional mechanism is not a prerogative of man or animals; it could be implemented in machines provided they posses an elaborate mechanism of creation.

algorithms depending on the attitude of the scholar. Von Neumann called these indeterminate, heuristic methods of play "good chess."[37]

M.Botvinnik[38] referred to the class of problems where such algorithms are used as inexact problems. Subsequently Botvinnik gave a more precise definition of the inexact problem. I quote it below.

> "We now say that an enumerative task is inexact if it solves a problem by minimax methods on a truncated search tree (see Glossary of Terms). The concepts of minimax procedure and search tree are well known; the concept of the truncated search tree may need explanation. Problems soluble by the formation of a tree of all possibilities (elementary actions) may differ in difficulty and may give rise to search trees of different sizes-small, large, or even infinitely large. If the resources of our information-processing device (speed and memory) are so limited that we cannot form the tree and search it exhaustively, we must either abandon the task or be content with an inexact (i.e., approximate) solution. If an inexact solution is acceptable, we limit the depth of the variations; the use of a depth-truncated tree and the acceptance of an approximate solution make the enumerative problem inexact. The definition of an inexact problem is therefore inextricably bound up with the general method of solution and with the resources of the information processing system being used. If we can apply the minimax procedure to the complete tree, the problem is exact. (Problems may of course be solved exactly by other methods, e.g, by the use of equations or exact algorithms, and they may be inexact under other definitions, as when approximate solutions to equations are used."[39]

§ 2. Algorithms of inexact problem - general description.

Generally speaking, final product, structure, function, process, and history, all being parts of a chess algorithm, must fulfill the following requirements:

Functions. a) to induce the opponent (for instance, influence opponent's choice between an aggressive and a defensive strategy).

b) to ensure at each stage of the game the capacity of one's position to adjust favorably to unforeseen moves.

c) to ensure at each stage of the game the capacity of one's position to absorb the impact of unforeseen moves if they happen to be disadvantageous to the player in question.[40]

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[37]Von Neumann, J. and Morgenstern, O., Theory of Games and Economic Behaviour, Princeton, 1944, p. 125.

[38]Botvinnik, M.M., Algorithm of a Chess Game, Moscow: Nauka.1968.

[39]Botvinnik, M.M., Computers in Chess: Solving Inexact Search Problems, Springer-Verlag: New York, 1984, p.1.

[40]Generally speaking, this approach to algorithm design differs from the adaptive approach which is based primarily on responding to changes in the environment by changing the internal organization of the mechanism of adaptation; here, only one of the functions mentioned above, namely the third one, is fulfilled. R. Ackoff suggests a new broader approach to the problem of adaption which is basically similar to my own. This is what R. Ackoff writes on this topic:

"To adapt is to respond to an internal or external change in such a way as to maintain or improve performance. The change to which adaptation is a response may present either a threat or an opportunity. For example, the appearance of a new competitor may present a threat; the disappearance of an old one, an opportunity. Both require an ability to detect changes that can or do affect performance and to respond to them with corrective or exploitative action. Such action may consist of a change in either the system itself or its environment. For example, if it suddenly turn cold, one can either put on additional clothing (change oneself) or turn up the heat (change the environment). Furthermore, the change to which adaptation is a response may occur either by choice or without it. The demise of a competitor, for example, may occur independently or because of what a corporation does.

Final product. In order to fulfill the above requirements we must know how to evaluate the final product generated at each step of the game. First, we shall describe the structure of the final product. The most difficult questions pertaining to inexact algorithms arise at this point. Future analysis shall focus primarily on the methods used in generating the step's final product.

Structure. The structure of the algorithm involves three dimensions: it has "length", "width", and "depth."

"Length" of an algorithm refers to its division into stages, stages into substages, and so on until we finally reach some primary step. The primary step (for instance in Hitech chess program) could represent a move chosen by each square of the board and indicating the best course of action leading to a potential placement of a piece on that square.

"Width" indicates the number of pieces included in the analysis of a given position. According to M. Botvinnik[41] the "width" factor can be split into three levels. (It seems to me that a fourth level can be introduced if the first level of Botvinnik's scheme is divided into two.) First level - identify the attacking piece. Second level - identify the stem pieces; these include the attacking piece along with all those pieces which, within the horizon of the attacking piece (same number of half-moves), help it achieve its goal. Third level involves the formation of a zone which M. Botvinnik defines in the following way:

> "Stem pieces do not act alone. Each is a crew of pieces of its own color who support it; there is also a hostile crew of pieces of the opposite color who hinder it. The ensemble of stem pieces and the two sets of commands of pieces of both colors from a field of play - and this is the second (intermediate) level of the control system ". (p. 34).

Fourth level deals with MM - mathematical mapping which includes several zones. It represents possible courses of action in the form of tree within which the actual search tree is constructed.

Finally, by "depth" I mean a hierarchy of programs. Besides the first-level programs, i.e. programs specifying what, how, from what, by which means, when and where, we must also have second-level programs which determine the first-level ones. There must also be higher level programs based on the experience derived from the game and the knowledge of the methods of solution of similar kinds of problems. The purpose of these programs would be to improve lower level programs including the first-level ones.

The concept of adaptation used here is much richer than the one used in association with the theory of evolution. In that theory, adaptation refers to only involuntary responses to external changes, and the responses consist of internal changes. This restricted connotation of the concept derives from the fact that the theory of evolution is preoccupied with nonpurposeful systems, and when it deals with purposeful systems it is not concerned with their purposefulness. Here we are preoccupied entirely with purposeful systems and their purposefulness". Ackoff, R., Creating the Corporate Future. New York: Wiley, 1985, pp. 170-171.
It seems to me that Ackoff's ideas on adaptation and their connection with the functions of an algorithm outlined above can be elucidated if presented in a more precise matrix-like.

Table 2. Methods of adaptation

Evaluation of the actual impact of the changes in the environment	Changes in the environ	
	Planned	R
Positive		
Negative		

[41]Botvinnik, M.M., The Cybernetic Goal of a Game, Moscow: Sovetskoe Radio, 1979.

Process. Methods of implementing the process can be grouped into several classes; these classes can be split into orders, orders into families and so on.

Here, I just want to note that the reactive principle of move selection invokes player response to a given situation that is the same all the time: "input" determines "output". This principle avoids the need to search through all the possible variations. In other words, the reactive approach means that the initial state determines the choice of the function which is known from previous experience. This function, in turn, transforms a given state into a new state. A search principle of selection shall mean the following: first, a player formulates a problem of finding the best move and then looks for it by searching through different variations.

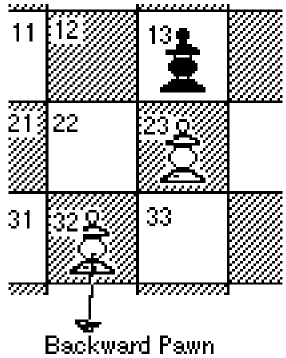
History: information about past play, both your own and your opponent's is important in choosing one's strategy (aggressive - defensive), in predicting the most probable moves, in establishing the value of a position, etc.

I want to repeat that every algorithm must fulfill the above requirements no matter what its implementation of the function, the final product, the structure, the process, and the history are. Attempts to make these general rules more concrete and finding ways of implementing them under a variety of circumstances has resulted in a number of different algorithms of play. Solution of the problem can be twofold - implicit or explicit. Under an implicit approach, all requirements are part of past experience represented in the form of specific and independent recommendations. Under an explicit approach, these requirements are incorporated in the formulation of an individual problem explicitly. Implicit and explicit approaches to inexact games are based on the reactive or the search principles of selection; naturally, these two principles can be combined.

§3. Reactive methods.

The typology of reactive methods introduced in the part of the book on large complex systems distinguishes between reflexes, instincts, images, and stereotypes. With regards to chess, a reflex is analogous to moving a piece away from a direct threat. An instinct corresponds to a situation where pieces are identified based on one single parameter, namely their ability to capture an opponent's piece. An image can designate such configurations as a backward pawn (see fig. 1) since it is bad on several counts: it can be captured, it creates a forpost for the opponents pieces which is protected by his pawn and which cannot be attacked by ones own pawns .

Fig.1



Stereotype refers to repeating the same move in a given position.

For my purposes, I shall distinguish only between a stereotype and three other reactive methods. Unlike a stereotype which encompasses a given situation in its entirety the other three methods take account only of some parts of a position. It seems to me that the widespread term "heuristic" is a good description of these three types of reactive methods.

Heuristics are valuable both for beginners as well as for advanced players. Naturally, heuristics' level of complexity varies depending on the level of play.

Having learned the rules of the game a beginner plays in a random like fashion. Having acquired a certain amount of experience, he begins to notice that some moves are good and some bad. Chess theory tries to formalize this experience and present it to the beginner in the form of recommendations. So the first steps in making a beginner's play more coherent consist of recommendations telling which moves are reasonable and even more importantly, which are not. These recommendations are similar in nature to heuristics. For instance, one of the first things learned by a beginner is not to place his pieces under attack or more precisely, not to blunder away his pieces; it is advisable to avoid making several consecutive moves with the same piece especially in the opening stage of the game, etc. Positive recommendations are also given; for instance, advance central pawns to gain control of the center and to develop the pieces.

The number of possible heuristics may grow with the level of skill. Some recommendations are valid only for the beginners. They are modified as far as more mature players are concerned. For instance, chess theory asserts that control of the center is not limited to its control by the pawns; hypermodernism introduced control of the center by opening up the flanks, i.e. by the bishops which assume extreme flank positions after a few preliminary moves. But hypermodernism requires considerably greater expertise on the part of the player.

I tried to point out a number of recommendations intended for beginners which contradict those given to the more advanced players in order to emphasize the contradictory nature of the heuristics themselves. We can assume that heuristics are given as independent statements. It is further assumed that these statements are always true. Some heuristics, such as "do not blunder" never violate these assumptions. But many heuristics are only true statistically which means that we can find a set of conditions under which they are not applicable. For example, there is a heuristic rule which disapproves of doubling one's pawns; yet there are cases where this rule does not apply (a player may want to use the open file for his heavy pieces). Moreover, heuristics may contradict each other. Therefore, they cannot be considered independently of each other in a given position.[42]

> "For instance, how can we make the heuristic "do not touch the pawns protecting the castled king" compatible with 'a very natural desire to drive away the opponent's trouble causing pieces or with the heuristic 'do not forget to make a window for your king'" ?"[43]

Conflicts between heuristics are not unusual.

> "Many heuristics used by the players are actually incompatible with each other. Therefore, we must create the same heuristic procedures to determine the importance of each heuristic. We then choose some subset of them depending on the nature of the position in question. This particular trend in heuristic design has seen very limited development."[44]

> Generally speaking, the reason for this situation with heuristics is the fact that chess theory as a whole is not based on one single concept but rather on a set of heuristics.[45]

From a methodological point of view, this means that recommendations-heuristics incorporated into an algorithm do not represent one single system: they are more like an assemblage (an aggregate) comprised of independent recommendations.

These independent recommendations comprising an aggregate are useful because of their statistical validity: they remain valuable even outside a given case. Each heuristic retains some "part" of its value even in situations where the heuristics are assigned priorities because their simultaneous use results in contradictions. The significance of the remaining "part" may become apparent in the future but at that particular point of the game its nature remains unclear. Therefore, situations which may give rise to contradictions among the heuristics should be avoided although in each specific case it may be possible to determine the priorities among the heuristics aposteriori depending on our goals and intentions. But this remaining "part" cannot be ignored in case contradictions among the heuristics are unresolvable and we have to resort to setting priorities among the them.

On the basis of general considerations we can say that aggregate structures comprised of a large and possibly growing number of elements are typical of many systems, especially in the beginning stages of their development. It is only after all the relations among the elements belonging to an aggregate have been uncovered that the latter turns into a system. Advanced systems may contain aggregates as well as subsystems (like systems, subsystems are also ordered). Only in certain extreme case and under certain conditions does an algorithm become completely free of all aggregates.

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[42]We have a similar situation with sayings and proverbs. Proverbs reflect wisdom and experience of many generations. But the unconditional nature of these sayings which sometimes results in their incompatibility makes it difficult to apply them in practical situations. For instance, a proverb "Two heads are better than one" has its opposite in the saying "Too many cooks in one kitchen". Two things must be done before we can apply these proverbs: we must either define conditions of their applicability or determine their priority in case their application to the same situation creates a conflict.

[43]Adelson-Velskii, G., et all, Machine Plays the Game, Moscow: Nauka, 1983, p. 70.

[44]Ibid.

[45]Ibid., p. 61.

Now consider stereotypes which are distinguished by their holistic approach to the position. Effectiveness of a stereotype is confirmed by previous experience and occasionally by theoretical considerations. Of course, there exist no rigid deductive proof of the impossibility of replacing a given stereotype with a better one.

Chess players' experience with stereotypes involves getting acquainted with the games of other players and learning the lessons to be used in playing similar positions occurring in one's own games.

In principle, this approach seems very conducive to the development of chess algorithms. According to C. Shannon, this methods involves having a "dictionary" of all possible chess positions. There is a "right" move (somehow arrived at or suggested by a grandmaster) associated with each position in the "dictionary." When it is machine's turn to move it just looks up the position and makes the appropriate move. The incredible number of possible positions - $64!/ 32!\, 8!^2\, 2!^6$ or approximately 10^{43} invalidates this approach.[46]

Obviously there are cases when algorithms incorporating previous experience (effective moves made in similar positions) are of value. The role of experience is especially prominent in grandmaster play and it can surely be implemented in machine algorithms.[47]

Still, it is hard to base one's play on remembering the best move associated with a given position because the number of possible positions is so large.

We should note in this connection that expert chess players are able to remember a rather large number of stereotypes since they memorize them in a very nontrivial way. Everything points to the fact that professional chess players memorize positions through their structure, i.e. groups of pieces making up certain configurations which H. Simon has called chunks. Psychological experiments conducted by A. de Groot were based on the hypothesis that chess player's memory is not purely mechanical. To test the hypothesis expert players and novices were given a position from a real game and allowed to look at it for five seconds. Some positions were quite complex with 20-24 pieces on the board. Professional players, master level and above were much superior to amateurs in this category. In random positions professionals and amateurs exhibited approximately the same abilities.[48]

It seems to me that expert players do not merely memorize a position but link it with the logic of possible continuations[49]. This leads me to conclude that an effective use of stereotypes calls for positional analysis characteristic of selective methods aimed at searching out novel solutions.

Moreover even if memory of the past is retained there is no guarantee that a given move is really the best one: it may be good; meanwhile best is the main enemy of the good. We would therefore like to be able to group different positions into classes, especially by

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[46]Shannon, G.,1950, "Programming a Computer for Playing Chess", The Philosophical Magazine, vol.XLI, 1950, p. 260.

[47]See Chess Skill in Man and Machine, ed. by P.Frey,1977, New York: Springer-Verlag.

[48]de Groot, A., "Perception and Memory vs. Thought: Some Old Ideas and Recent Findings", Problem Solving, ed. by B. Kleinmuntz. New York: Wiley, 1966, pp. 19-50.

[49]This reminds me of poets who remember a lot of poetry thanks to their understanding of the laws of poetic construction and remembrance of original key words which allows them to reconstruct the flow of a poem. Poetic process of reconstruction may proceed very fast and remain unnoticeable to an outside observer. In other areas this process is far slower. For instance, in mathematics one can memorize a formula or the logic of reasoning behind a whole class of formulas. The latter method may require substantial amount of time in order to come up with a specific formula.

using chunks. Differences among positions belonging to the same class would be so minor that a move which proved to be good in one of them in the past can be played in all the positions belonging to that class. But it is rather difficult to define all the criteria to be used in deciding whether or not a particular position belongs to a given class. These problems are well known in psychology in the theory of gestalts and in artificial intelligence as it pertains to pattern recognition. Here selective methods come to the rescue. These methods are typically aimed at searching out new continuations from a given position. This approach requires a holistic evaluation of a position since there must be a criterion to judge which of the possible positions is best. Note that different positions being close in value does not mean that they call for the same methods of play. It merely indicates that they have equal predisposition towards future development.

§ 4. Search solutions.

We shall start by examining various types of optimization algorithms. This will be beneficial in subsequent application of these algorithms to inexact problems as well as in finding their most pragmatically feasible subclasses. In some extreme cases when parts of an inexact algorithm can be reduced to an exact problem these methods can be applied in their entirety.

Theoretically there are three different methods of constructing an optimization algorithm. The first method involves representing the entire game as a system of equations (inequalities) and then finding the values of the variables which correspond to the optimal value of the objective function. But representing a chess game as one single system of equations (inequalities) linking its initial state with the final goal seems an impossible task.

The second method of solving optimization problems is to

> "... to replace the actual problem space with a very much smaller space that approximates the actual one in some appropriate sense, and then apply the classical theorem to the smaller approximate space."[50]

This means that a problem like chess is to be broken down into a series of elementary parts with each part assigned a local problem whose solution will ensure a global optimum. The major idea behind this approach is the following: on the one hand, each local problem is given some degrees of freedom (choice) within the framework of the respective methods of transforming inputs into outputs. On the other hand, each local problem cannot be linked directly with the final, global goal associated with the problem as a whole (except for the final local problem); also the constraints of each local problem (except the first one) derived from the solution of a preceding local problem are not known. Therefore the formulation of a local problem requires some indirect information to be used in defining its local criterion of optimality. This criterion must guide the process towards a global optimum for the system as a whole. The criterion must also be implemented in a condensed form so that the local problem does not become too cumbersome and unsolvable for a given operator.

[50] Simon,H., "Theories of Bounded Rationality", Models of Bounded Rationality, vol.2. Cambridge: The MIT Press, 1982, p. 413.

Methods for solving these types of problems are known as algorithms of dynamic programming and decomposition principles of dynamic programming. These algorithms initiate search for the optimum point at the end of the game and gradually, using recursive procedures work their way to the the initial state. This approach ensures that the arrived at local solution is definitely an optimal one as far as the "future" is concerned since the procedure is structured "backwards" from the "future" to the "present." Nevertheless, methods of dynamic programming have limited application being of practical use only for problems having a small number of variables.

Other search algorithms for solving inexact problems are similar to the methods of dynamic programming: a gradual, step by step solution of a local problem coupled with a criterion for evaluating the "strength" of a given position and the constraints generated by the solution of the problem at the previous iteration. One major difference between these search algorithms and dynamic programming is the direction of motion which is, in the former case, from the "present" to the "future."

But this method does not take the game to completion or even to some intermediate stage - the horizon - which ensures success for this would, in the opinion of some chess players require looking 15 moves ahead especially in the middle game. This is the source of uncertainty as far as solution of a local problem leading to the desired outcome if one starts at the "beginning".

Especially interesting in this respect is the mathematical concept elaborated by Atkin.[51] Atkin's concept hypothesized about solve dynamic problems using algorithms which proceed from "beginning to end". I know of no attempts to apply this concept to chess.

Finally, I want to examine the so called decomposition methods of optimization. Basically, what makes them different from dynamic programming is that each local problem can be decomposed into sublocal problems which can be broken down further. Hierarchical organization of problems allows for completely new methods to be introduced: solution of local problems belonging to the same level can proceed in a parallel fashion while local problems belonging to different levels are related to each other in a sequential manner.

The idea of organizing local problems in a hierarchical fashion is used rather extensively in inexact chess algorithms. But here, unlike decomposition methods of optimization, hierarchy does not provide for parallelism. Rather it is used to set up some kind of an intermediate goal within a sequential process. This helps coordinate sublocal problems which are also structured sequentially. Hierarchy is also used in breaking up a sequential process of play in order to classify some aspects of the game (more on this below when we analyze different stages of the game). Judging by the brief description of chess program Hitech[52] its algorithm incorporates the idea of decomposition but it is limited to one move. At every move each square that is potentially a landing ground for some piece suggests to the "coordinator" what it thinks is the best alternative from among the ones leading to the placement of a piece on that square. The coordinator in turn hands down its evaluations of actions to each individual cell. The process is executed in a parallel fashion.

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[51] Atkin, R., 1972, "Multi-dimensional Structure in the Game of Chess", Int. J. Man-Machine Studies, 4, pp. 341-362; Atkin, R., 1975, "Multi-dimensional Approach to Positional Chess", Int. J. Man-Machine Studies, 7, pp. 727-750.

[52] See Berliner, H., and Ebeling, C., "The SUPREM Architecture: A New Intelligent Paradigm", Artificial Intelligence, 28, 1986, pp. 3-8.

Our subsequent discussion of search methods used in the development of "inexact" chess algorithms is based on well-known chess programs having a rigid sequential organization of the local problems. Even if these algorithms do utilize a hierarchical principle it is supplemented by a sequential analysis of the problem.

Taking into account one single aspect of the problem, namely the multi-stage nature of the process, leads to the following question: "Should the local problem be uniform throughout (for all the moves it is supposed to handle) or should it vary?" Experience has shown that the set up of a local problem which is being solved as part of a global problem can vary. It is beneficial to vary the structure of a local problem since this makes for more effective solutions. At the same time, the solution of the global problem becomes more complicated. A heterogeneous algorithm, unlike a homogeneous or a unified algorithm which handles only one type of local problems, needs additional information from the local problems to ensure their subsequent integration into a single whole.

No matter what the set up of the local problems is we can never avoid the general question of the value of a given position in terms of its impact on the future of the game.[53] In fact, it will become clear in our subsequent analysis that evaluation of the current position is crucial in setting up local goals even in combinational style of play.

Analysis of a state, i.e. current position in the game, is vital. Such analysis should aim at establishing the proper set of parameters and their respective weights which should enable us to express the integrability of a given position with the game as a whole.

Moreover, there exists a trade-off between depth of search and complicated weight function.

> "Extrapolating from several microcomputer implementations at moderate speed with correspondingly constraint depths of search and complexity of evaluation function, the author is lead to conjecture that there is no difference - between deep search and superficial evaluation, and shallow search and complicated evaluation, though deeper search may provide stronger resistance to human opponents."[54]

These considerations form the foundation of our analysis of the structure of a position. Let parameters determining the value of a position be called parameters of integration. The next chapter examines these parameters in greater detail.

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[53]"The problem confronting a chess player whose turn it is to move can be interpreted in either of two ways. First, it can be interpreted as a problem of finding a good (or the best) strategy - where "strategy" means a conditional sequence of moves, defining what move will be made at each successive stage after each possible response of the opponent.

Second, the problem can be interpreted as one of finding a set of accurate evaluations for the alternative moves immediately before the player.

From a classical standpoint, these two problems are not distinguishable. If the player has unlimited computational power, it does not matter whether he selects a complete strategy for his future behavior in the game, or selects each of his moves, one at a time, when it is his turn to play. For the way in which he goes about evaluating the next move is by constructing alternative complete strategies for the entire future play of game, and selecting the one that promises the best return." Simon,H., "Theories of Bounded Rationality", Models of Bounded Rationality, vol.2. Cambridge: The MIT Press, 1982, p. 412.

[54]White, J. F., "Querg Chess", ICCA Journal, Vol. 11, No. 2/3, 1988, p.72.

Chapter 3. Evaluation of a state: parameters of integration.

§1. Analytical representation of the holistic effect.

Evaluation of the current state must reflect the state's overall impact on the game. But does the holistic effect yield to an analytical representation, i.e. can it be expressed by means of independent evaluations of different parts of a position? Proceeding in a straight forward manner we would have to assume that all parts are homogeneous and have the same impact on the environment, i.e. they have the same weight (importance, value). Under these assumptions individual parts can simply be added together (two apples plus three apples equals five apples). The value arrived at represents one of the essential preconditions for further development. But suppose different parts are qualitatively different or, if they are the same, have different "weights." How then do we integrate them into a single whole? This can be accomplished if we take into account the "weights" of the parts (two apples weighing 80 grams each plus three apples weighing 100 grams each together weigh 460 grams; this characteristic is essential for further development). Determining (discovering) these weights is a rather difficult task. Physics has had the most success in this area. The situation with biological systems where these weights are represented by, say emotions, with economic systems where they assume the form of prices, and with a number of other fields is more complicated.[55]

So it is possible to express the holistic value of a given state through its parts analytically provided their integration proceeds by calculating their respective weights which reflect the marginal contribution of each part to the development of the whole. This is the way many mathematical procedures for solving optimization problems are structured. Condensed, global evaluations of different parts, namely the Lagrange multipliers generated by these procedures, are precisely these kinds of weights. They are global in a sense of allowing us to measure the marginal impact on the overall development of the system produced by independently varying one single variable at each local step. (Since chess pieces are discrete in nature and are rather small in number the idea of varying one single variable must be reconsidered). The global nature of Lagrange multipliers is due to the fact that the process by which they are formed takes all the constraints of the problem into account: the initial state, the final goal, and the rules of the "game".

The sum of the products of each component with its respective global value represents the level of system's development at a given stage; it measures the impact of the current state on the solution of the global problem.

If we look at chess from this perspective we can say that it is possible to give a global evaluation of a position in analytical terms at each local stage provided we know an exact algorithm of the game. This evaluation would reflect the impact of a given position on the outcome of the game. But what if such an algorithm is not available, what if there is no

[55] The difficulty of determining the exact relative weights of all the parameters lead to an attempt to simplify the problem in the following way: to try to discover one dominant part and use it to evaluate the impact of the entire state on the future. (In sciences, this idea is known as using one single characteristic to determine the overall state). But such dominant characteristics are rather rare.

program linking parts with a whole in a rigid and precise fashion, how then do we determine the holistic effect of a given position?

May be we can try to apply the ideas of exact algorithms to determine approximate values of parameters used in evaluating the current state of the system. These values should be a good approximation of the weights generated by exact algorithms. Of course, it should be remembered that the problem of evaluating the overall position is the cornerstone of our analysis. Evaluation of position's individual parts or of a single piece remains an important issue as far as varying their values independently of each other, but it is essential to keep in mind the interrelationships between the parts and the whole.

What does all this mean? It means that we have to proceed from the general to the particular, i.e. from an overall position to individual pieces.

Two questions arise immediately: 1) how do we analyze the structure of a position, i.e. will it be sufficient to limit our information to such primary objects as pieces - material (together with their coordinates) or will we need additional primary objects? and 2) how do we measure the values of the pieces when we do not know their total worth relative to some final goal?

Affirmative response to the first question gives rise to two new questions which reveal the need to introduce new primary parameters to be used in the evaluation of a position. The two questions are 3) what is the role of this new set of primary parameters and 4) how to evaluate these new parameters so that they are commensurable not only among themselves but also with the weights of material objects which together provides us with a evaluation of the position as a whole?

Let us proceed to answer these questions.

The first question asks whether the information pertaining to such primary objects as material parameters is sufficient to evaluate a position. The answer to this question bears directly on the second (as well as third and fourth) question which deals with evaluation of the material in inexact algorithms. I want to repeat that the structure of inexact algorithms rules out the possibility of linking the current state with the end of the game, i.e. of deducing the overall values of the pieces which would be sufficient to evaluate a position. We are thus forced to take a different route: first we assign unconditional values to the pieces, i.e. values corresponding to some ideal situation. We then try to introduce additional parameters which are supposed to reflect the current situation on the board, its specificity. This really represents an attempt to make up for the incompleteness of evaluations by including unconditional weights of the pieces. Now evaluation of these additional parameters which could be called compensatory parameters must account for their impact on the future flow of the game. Here parameters used to compensate for this deficiency pertain not only to individual pieces but also to groups of pieces or to the position as a whole. It seems to me that the nature of these parameters lies in the interrelations among the pieces in a given situation. It is precisely these relations which were not incorporated in the unconditional weights of the pieces. The more complete this set of parameters and the better their weight assignment the closer (other conditions being equal) is our evaluation of a position to that of an exact algorithm.

Therefore lack of information on the total value of the pieces (this information is needed to evaluate a position) can be compensated by determining the value of the entire position by means of unconditional values assigned to the pieces coupled with the parameters characterizing the relations among the pieces.

The next section explores the first component of the problem.

§ 2. Evaluation of material

Our preliminary discussion will focus on the analytical construction of unconditional values of the material. One reason for this is the availability of literature devoted to this topic. Another reason is the uniqueness, as I see it, of chess in its approach to determining the weight of the material.

Traditional treaties on chess - those devoted to the theory and practice of the game, are not concerned with the analytical methods of calculating the unconditional values of the pieces. For instance, E. Lasker's now classical chess manual does give us the unconditional weights of the chess pieces. The ratios presented in the above text are well known to all chess players. The author points out that these values are valid only in equal positions. The ratios between the values of the pieces can vary considerably depending on the specific situation on the board. Just like many other players Lasker believes practical experience to be the source of the weights assigned to different pieces. The following quote from Lasker's book not only confirms my statement but also presents some possible relative numerical values of the pieces. These values will be important to us in the future.

> "...the masters, and among them certain regularities show very plainly. The experience derived from games played by such as deserve the title of masters, during centuries has proven these regularities."[56]

In his concluding remarks which really stand apart from the rest of the book Lasker gives us an interesting comparison between the values arrived at on the basis of practical experience and those produced by the mathematical analysis of the game.[57] He points out that Euler who was a great mathematician but a rather poor chess player was able to determine the values of the pieces mathematically and these values happened to correspond closely to the values representing the practical experience of great players.

The procedure used by Euler to determine the values of the pieces is not known. But Lasker points out that Euler's method "bears the seed of future discoveries"[58] Lasker does not give reference to Euler's work which could have been used to reconstruct his chain of reasoning using our present-day knowledge of the subject.[59]

Judging by Lasker's comments it is reasonable to assume that Euler used rather static positions to compute the values of the pieces, i.e. a case where material plays a decisive role. Two beginners playing in a random like fashion start from a given position and play n-games (n approaching infinity); material advantage for either side is determined by the winner of the match.

Lasker also notes that for Euler the mobility of a piece was probably the dominant characteristic in establishing its material value. Mobility, in turn, is determined by the free

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56 Lasker, E., Lasker's Manual of Chess, Philadelphia: Mc Kay Co, 1947, p. 35.

57 Ibid., pp. 340-341.

58 Lasker, E., Lasker's Chess Manual, Moscow: OGIZ, Fizkultura i Turizm, 1937, p.323.

59 My own attempts to find this work were unsuccessful. I even asked specialists on Euler from the East German Academy of Sciences but they were unable to help me either.

space or by the "obstacles" (thus a knight, although it is less mobile than the bishop, has an advantage over the bishop in being able to jump over the "barriers" set up by the opponent).

Well-known attempts at elaborating analytical methods to calculate values of the pieces were undertaken in non-chess literature.

Results derived from these non-chess sources coincide with practical experience to a surprising extent. This analysis is very valuable since it reveals the reasons why pieces have different weights. The weakness of these results lies in the fact that practical experience probably incorporates other important factors which may cancel each other.

The groundwork in the field of unconditional evaluation of the pieces (which happened to coincide with practical experience) was an article by H. Taylor.[60] In this work (I shall quote from it without further reference) Taylor proposed two methods of measuring the relative material values of the pieces. Common to both of these methods (this goes for Euler's method as well) was the dominant role of the material. Taylor wanted to reduce his experiment to the simplest possible case: the only pieces on the board were a king for one side and one piece for the other (besides king and pawns). Each piece wants to check the opponent's king. This is the set up of the problem:

> "A king and a piece of different colors are placed at random on two squares of a chessboard of n^2 squares: it is required to find the chance that the king is in check."

Unlike Euler's experiment, this method of calculating the relative values of the pieces for a given size board is based on a finite number of trials since the number of squares that can be occupied by a king and one piece is limited by the size of the board.

Taylor discovered that his method of calculating relative weights of the pieces produces results different from those used by the players. So he modified his experiment. Instead of calculating the probabilities of king checks he introduced an additional requirement of a "safe check." "Safe check" differs from a "simple check" in that the piece making the check cannot be captured by the opponent's king. The notion of a "safe check" incorporates two aspects: the mobility of the piece plus its ability to "overcome obstacles." In order to determine the relative weights of the pieces which E. Lasker spoke about Taylor elaborated a formula to calculate the probability of a king being in check, both "safe" and "simple." Table 3 below is borrowed from Taylor's article. It shows the results of his experiment for any size board having an even number of squares and for a regular board having 64 squares. Here, **n** stands for the number of squares on the board.

Ordering (reducing to a common denominator) the probabilities of different pieces putting a king in check gives us the following sequence: for the case of a "simple check" the values are 3, 5, 8, and 13 for the knight, the bishop, the rook, and the queen respectively; for the case of a "safe check" they are 3, 3.25, 6, 9.25 (12, 13, 24, 37). The correspondence of the last set of figures to the ratios commonly used by the players (they are 3, 3, 5, 9) is amazing.

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[60] A popular presentation of this work by Taylor is given together with some commentaries in chapter 6 of Rouse Ball, W.W., Mathematical Recreations & Essays. New York: The Macmillan Co,1962, pp.161-165.

Table 3. The probability of a king being in check

Chessboard with n2 squares			
	Normal	Safe	
Knight	8(n-2)/n2(n+1)	8(n-2)/n2(n+1)	1/1
Bishop	2/3x2n-1/n (n+1)	2/3x(n-2)(2n-3)n (n+1)	5/36
Rook	2/n+1	2(n-2)/n(n+1)	
Queen	2/3x5n-1/n(n+1)	2/3x(n-2)(5n-3)n (n+1)	13/36

Working within an even more general setting M. Kavesh suggested evaluating a pawn by the number of safe checks it can give from the eighth rank where it can be promoted to a queen. The results coincided with the established value of the pawn relative to other pieces.

Attempts independent of those by Euler and Taylor to determine the relative values of the pieces analytically were made in more recent times. The basic idea was to have a clear board with one single piece. Then an average number of moves that a piece can make depending on its position on the board was calculated. For instance, a rook can make 14 moves from any square on the board. Obviously, an average number of moves it can make is also 14 (14x64/64). A queen can make 21 moves from 28 squares of the board, 23 moves from 20 squares, 25 moves from 12 squares, and 27 moves from 4 squares. Altogether a queen can make 1456 moves (21x28+23x20+25x12+27x4). Its average mobility is 22.75 moves (1456/64). Similar calculations can be performed not only for the knight and the bishop but also for the pawn and the king. Thus, a knight can make an average of 5.25 moves ((2x4+3x8+4x20+6x16+8x16)/64), a bishop can make 8.75 moves ((7x14+9x10+11x6+13x2)/32). As an autonomous piece, a king can make 6.5625 moves on the average ((3x4+5x24+8x36)/64); a pawn can make an average of 2.5 moves ((2x10+3x32+4x6)/48). Here, it is assumed that a pawn cannot occupy the first rank or move backwards from the eighth rank and the possibility of its promotion upon it reaching the eighth rank is ignored.

Below we have a formula for calculating an average number of possible moves. They are denoted by P_n for all the pieces[61] and the size of the board which is arbitrary is denoted by n.

$P_n(K)=4(n-1)(2n-1)/n^2$

$P_n(Q)=2(n-1)(5n-1)/3n$

$P_n(R)=2(n-1)$

$P_n(B)=2(n-1)(2n-1)/3n$

$P_n(Kn)=8(n-1)(n-2)/n^2$

$P_n(P)=(n-1)(3n-4)/(n-2)$

Taking pawn as a basic unit of measurement the relative values of the pieces are 2.1 for the knight, 3.5 for the bishop, 5.6 for the rook, and 9.1 for the queen. These ratios in the values of the pieces are very close to those worked out by Taylor who used the "simple

[61]Derivation of the formula for the bishop is rather complicated so Gik recommends a book by Okunev, L. J. entitled Combinatorial problems on the chess board, Moscow: ONTI, 1935.

check" criterion (values arrived at by Taylor must be divided by 1.428 to even them out with the ones generated by the above method). This fact is not a coincidence. Indeed, the formula for king checks differs from the formula for the average number of moves that can be made by a given piece in only one respect: the former method takes the average number of possible moves available to a given piece relative to the total number of squares on the board and the latter takes the sum total of all possible moves and divides it by the total number of squares on the board.

An example with a rook should help illustrate the point. Using Taylor's method (the "simple check" criterion), the number of checks that a rook can make is 14 no matter what the position of the king is, i.e. 2(n-1). On the average, a rook will be able to do so from (n^2-1) squares since one of the squares is always occupied by itself. Therefore, the probability of a rook check is 2(n-1)/(n^2-1)=2/(n+1). This formula differs from the one which calculates the number of possible moves 2(n^2-1)/(n+1)=2(n-1) by a multiple of (n^2-1). In other words, in calculating the overall probability of a check that a rook can make from a given square we discover that it is precisely 2(n-1). The same considerations apply to cases where the number of possible checks varies depending on the position of the piece. A reader can perform these exercises by putting (n^2-1) in the numerator of each of the formulas. A discrepancy between the value of (n^2-1) and 1.428 - a coefficient used to compare the values of the pieces calculated according to one of the aforementioned methods, arises as a result of the chosen standard (a particular unit to measure the relative values of the pieces).

Although we arrive at the same relative values using either the Taylor's formula for "simple checks" or the mobility of the piece formula, the assumptions inherent in these two approaches have very different implications. Taylor's formulation of the problem is more meaningful: it relates the value of the piece with the final objective of the game. This simplifies our task of finding the appropriate set of constraints to be incorporated into our experiment aimed at establishing the relative weights of the pieces and make our analytic results resemble those arrived at on the basis of practical experience. The other more abstract approach which is based on piece mobility does not guarantee affinity of the results. Still, it is advantageous in some respects; for instance, it allows both the king and the pawns to be evaluated as active pieces.

To sum up we can state the following: the relative values of the pieces are functional in that they reflect the potential contribution of a given piece to the final goal of the game. At the same time, these values are idealized, i.e. they are deduced under "perfect" conditions practically unattainable in the real game. These values are rather unusual being averaged out and completely unconditional. The choice of the averaged out unconditional values rather than the highest ones generated by the most favorable circumstances "softens" the unconditional nature of these values. It makes the game more concrete. For instance, it takes into account the possibility that in some distant future a king and one piece may come to be positioned on any squares of the board.

This approach to evaluating the pieces leads to conclusion that the value of a piece is always the same. To make its value more concrete, a piece has to be examined within the context of a specific position. This is where positional parameters come in. For instance, some players claim that the values of the pieces in the opening are different from the usual rating. This means that the values of the pieces change as a result of positional considerations. E. Lasker writes that in the opening the relative values of the pieces differ depending on their initial positions.[62] They also differ from the commonly accepted weights of the pieces. The second column of Table 4 gives the values of the pieces according to Lasker; other columns compare his values with the commonly accepted ones.

Unfortunately, Lasker does not discuss any analytical methods for generating the values of the pieces in the opening. Still, it is easy to see that one of the major factors responsible for making the opening values of the pieces different from their overall ones is the immobility of the pieces in the opening, the fact that some pieces limit the development of others.

It would be very interesting to do an experiment to represent the modified (Lasker's) values of the pieces in the opening analytically by introducing a number of positional parameters. I leave this exercise to the reader.

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[62]Lasker, E., Lasker's Manual of Chess, Philadelphia: Mc Kay Co, 1947, pp.106-107.

Table 4. A comparison of the values of the pieces in the opening (suggested by Lasker) with the established values.

The piece	Its value in the opening	Averaged out value opening: by type of piece	Averaged out value opening: rel. to a pawn	Averaged out value general	The difference between relative averaged out value in the opening and overall: absol.	%
Pawn: **d** or **e**		2	1.3	1	1	0 0
" **c** or **f**	1.5					
" **b** or **g**	5/4					
" **a** or **h**	1/2					
Knight	4.5	4.5	3.5	3	+0.5	+16.7
King bishop	5	4.75	3.65	3	+0.65	+21.6
Queen bishop	4.5					
King rook	7	6.5	5	4.5	+0.5	+11.1
Queen rook	6					
Queen	11	11	8.5	9	-0.5	-0.6

Concluding our discussion on evaluating material parameters, I want to repeat that our ability to deduce these values is largely due to the peculiar set up of the game discussed in the first part of the book. The only point to add to that discussion is that every piece in chess except a king can pose a direct threat to enemy king even if the piece itself is "not in danger," i.e. every object in chess has the capacity to be directly involved in achieving the final goal. This applies to pawns as well for they can be promoted to stronger pieces enabling them to fulfill the above mentioned function.[63]

§ 3. The structure of positional parameters

I hope the above analysis of the methods of forming unconditional material values has made it clear that other parameters have to be introduced in order to improve upon our unconditional values in each specific situation. Doing so will enable us to give an overall evaluation of the current position (or some possible future position). These additional parameters must reflect interrelations among the pieces. As I have already mentioned above the purpose of introducing these parameters is to do explicitly what an exact algorithm of the game does implicitly.

I want to start our discussion with primary positional parameters, i.e. parameters that cannot be decomposed any further with the tools presently available. Primary parameters can be grouped together to form aggregate positional parameters such as the strength of the center, the development of the right or the left flanks, and so on. I shall not touch upon this kind of parameters in the present work. I only want to note that a lack of research on the methods of aggregating-disaggregating chess positions represents a gaping hole in the field of computer chess. There is no doubt that good chess players utilize these kinds of procedures to a great extent.

[63]All this leads to a very interesting question which still remains open: "To what extent can the methods used in chess to form unconditional weights of the pieces be used as a general principle in other systems having some universal intermediate objects, i.e. objects not directly related to the final goal even if such a goal does exist?"

Beginning with Steinitz chess players have paid a lot of attention to primary positional parameters. Examples of these parameters are scattered throughout E. Lasker's chess manual. For instance, on page 202 he examines such positional parameters as double pawns, backward pawns, and isolated pawns. Lasker also mentioned a number of positional parameters that have not received wide acceptance till this day. He stressed the importance of positional parameters usually associated with pawn structure in analyzing piece coordination. Elaborating upon Steinitz's ideas, Lasker points out

> "If, one the one hand, the distance of two pieces, for instance of two knights, is very great, a co-operation or a conflict between them is possible only in the future, unless they co-operate or contend with each other indirectly..."[64]

It seems that the first systematic list of positional parameters was put together by C. Shannon.[65] As far as I know, it is still the only systematic list of positional parameters to be found in the chess literature; therefore, it is given here in its entirety.

"(2) Pawn formation:

(a) Backward, isolated and doubled pawns.

(b) Relative control of center (pawns at K4, Q4, B4).

(c) Weakness of pawns near king (e.g. advanced KNP)

(d) Pawns on opposite color squares from bishop.

(e) Passed pawns.

(3) Positions of pieces:

(a) Advanced knight (at K5, Q5, B5, K6, Q6, B6), especially if protected by pawn and free from pawn attack.

(b) Rook on open file, or semi-open file.

(c) Rook on seventh rank.

(d) Doubled rooks.

(4) Commitments, attacks and options:

(a) Pieces which are required from guarding functions and, therefore, committed and with limited mobility.

(b) Attacks on pieces which give one player an option of exchanging.

(c) Attacks on squares adjacent to king.

(d) Pins. We mean here immobilizing pins where the pinned piece is of value not greater than the pinning piece; for example, a knight pinned by a bishop.

(5) Mobility." (p. 274).

Authors of chess program "Caissa" came up with a more elaborate but still not systematized set of positional parameters. The first version of the program included 18 parameters.[66] Subsequently the list was increased to 31.[67] Classification of the parameters

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[64]Lasker, E., Lasker's Manual of Chess, Philadelphia: Mc Kay Co, 1947, p.239.

[65]Shannon,G., "Programming a Computer for Playing Chess", The Philosophical Magazine, vol.XLI, 1950.

[66]Adelson-Velskii, G., et all, "Programming the Game of Chess", Soviet Mathematical Proceedings, v. XXY, issue 2/152, 1970.

[67]Adelson-Velskii, G., et all, Machine Plays the Game, Moscow: Nauka, 1983.

included in this list is rather simple. First of all, 12 parameters associated with squares and pawns were distinguished from those associated with squares and pieces. The authors had also come up with 10 parameters reflecting the dynamics of the game. Among these parameters were "attacking the opponent's strongest piece," "a light piece attacking a weak square in the opponent's position," etc.

This is what the authors wrote concerning the subject:

> "...at first, a rook must attack a square lying on an open file, then it must occupy it, and finally "invade the gluttony row". The opponent, in turn, must try to prevent this from happening by creating a position devoid of such weaknesses."[68]

And finally they distinguished another very specific parameter "Bishop attacking any piece belonging to the opponent (except a pawn)" which is really a particular case of another parameter. A parameter called "Nail" which describes a particular piece and pawn formation was also introduced.

I want to propose another classification of positional parameters presented in the aforementioned works on the "Caissa" chess program. It is given in Table 5. I believe this classification is more systematic. Its matrix representation reveals the connections between the relations and the objects comprising them. Pawns, because they are rather special (for instance, they have less mobility than other pieces and form relatively static structures - the skeleton of the game), are allotted a special place in this classification.

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[68] Ibid., p.55.

Table 5. Classification of positional parameters

Types of the relations	Object of the relations					
	Squares	Pawns and pawns	Squares	Pieces	Piece and pawns	Squares,Pieces and pawns
Configuration	12.Open file****	2. Phalanx 6. Isolated Pawn	3.Center pawn* 5. Passed pawn 9.Isolated pawn on a semi-open file*****	32. 'Nail'^^^^ Pin^^^^^	19. Light piece occupying opponent's weak square 20. Knight posted in the center 22. Queen (rook) on an open or a semi-open file 23. White rook on the seventh or eight rank++++	(2)(c) weakness of the pawns near king∆
Control	10. Weak squar**	4. Pawn attacking a center square		14. Scope 15.Opponent's highest ranking piece under attack++ 16. Opponent's unprotected piece under attack+++17. Fork++++ 31. Bishop attacking any opponent's piece (except a pawn)	+18. Light piece hitting opponent's weak square 21. Rook (queen) on an open or a semi-open file 24. A square next to the opponent's king under attack 25. Queen-king scope++++++ (4)(b) attacking pieces with possibility of exchange	
Quantity and variety	number of free squares					(3)(g) pair of rooks^^^ variety of pieces bishop&rook vs. knight&rook; queen&knight vs. queen&bishop; two bishops vs. two knights^^^^
Loosing opportunity to perform a special operation					27. King castled 28. King-side castle given up° 29. Queen-side castle given up°° 30. Castle given up°°°°	

Comments to Table 5.
Arabic numerals refer to the list of parameters taken from the book by Adelson-Velskii, G., et all,1983, Machine Plays the Game, Moscow: Nauka. If the Arabic numeral is accompanied by a letter index and both are in parenthesis, then they refer to the parameters taken from the 1950 work by C. Shannon. No number in front of the parameter means that it was borrowed from other sources referred to separately.
Upper indices indicate that the respective parameter is accompanied by a footnote; I introduced all these symbols only for the sake of convenience. The need for explanations arose because players and programmers sometimes use different terms for the same parameter and some positional parameters are expressed in chess jargon which may be unfamiliar to the reader.
Explanation of the parameters:
*For white - squares e4, e5,e6, d4, d5, d6; for black - squares e5, e4, e3, d5, d4, d3.
**"(Hole) - a square under attack by an opponent's pawn and is not and cannot be defended by ones own pawns unless it moves to a different file as a result of a capture.)" -
***"(Weak pawn) - a pawn standing on a square that lies behind a "hole".
****Open file
*****File open to a given player - only opponent's pawn(s) on the file
+ "For each piece, including a pawn, the squares that come under its attack including those occupied by its own pieces or pawns (squares it defends)."
++"A piece occupying a square that is under attack by an opponent's piece(s) but not defended by one's own piece(s)."
+++"A piece occupying a square that is under attack by an opponent's piece(s) but not defended by one's own piece(s)."
++++"(Fork) - opponent's piece attacking two undefended higher ranking pieces of a given player."
+++++ Black rook on the first or siccing rank.
++++++ "A square that can be occupied by a queen if it was placed on a square currently occupied by the king. Having occupied it, opponent's queen (from vertical and horizontal squares also a rook, bishop from the diagonal squares) gives your king a check."
° A rook occupying the h1 square in the starting position has moved
°° A rook occupying the a1 square in the starting position has moved
°°° Both rooks or a king have moved
°°°° "Any piece or a pawn in front of the opponent's pawn occupying its original position between the c and the f files. This structure splits the flanks of the opponent."
°°°°° A given player must protect the piece in question with another piece
^ For instance, an advanced king night pawn
^^ Here, they mean a "pin" where the value of the pinned piece does not exceed the pinner; for instance, a bishop pinning a knight.
^^^ Variety of pieces, especially important in combinational play since it creates more opportunities.
^^^^ According to Capablanca, a bishop and a rook are stronger than a knight and a rook, but a knight and a queen may turn out to be stronger than a bishop and a queen; two bishops are almost always stronger than two knights.[69]

[69]Capablanca, H-R., Chess Manual, Moscow: Fizkultura i Sport,1975, pp. 30-31.

I believe by now the reader has sufficient empirical material to understand the diversity of positional parameters.

I want to make one additional remark. It is very unclear whether the above set of positional parameters is exhaustive. First of all, the language of the chess players is rich but it lacks precision. Therefore, just the formalization of parameters in chess players' arsenal presents certain difficulties: even qualified chess programmers are sometimes unable to do this.[70]

Assuming these difficulties can be overcome, there are other more serious problems. We can look at objects from an infinite number of perspectives so the number of positional parameters also seems to be infinite. Choosing a subset of positional parameters relevant to a given problem still remains an unresolved issue.

It is close to impossible to construct a system from which we can deduce the entire set of positional parameters which would be sufficient to create an "invincible" algorithm of the game.

Unfortunately, the set of positional parameters currently used by computer programs is still based on the experience of good players who express it in an intuitive manner. I think that one of the major obstacles to designing more effective algorithms is a lack of formalized procedures for developing new positional parameters. These procedures lie at the foundation of second-level programs, i.e. programs that modify other programs directly responsible for generating the moves.

Finally, let us consider the problem of evaluating positional parameters since their share in the overall value of a position may vary. I know of no analytical works on this subject that would even resemble analogous works on the evaluation of material parameters. In fact, I am familiar with two works which contain intuitive evaluations of positional parameters worked out by expert players.[71]

Whereas the first work presents only intuitive evaluations of positional parameters, the second one elaborates a number of ideas on how to form them. In Slate's work the general idea of assigning values to positional parameters (as well as material ones) is founded upon the following assumption:

> "Basically, the function adds several easily computable factors together. The dominating term is material. The remaining terms are rules of thumb designed to give the program a vague positional sense. Rather then expressing some theoretically rigorous chess principles, these factors merely improve somewhat over 'aimless' moves. The design motive for the evaluator appears to be a self-fulfilling prophecy: we knew that it was going to be primitive and would thus depend heavily on tree searching. To do such deep tree searching would require that the evaluator do its work quickly, and so it wouldn't have the time to do anything clever, and thus would have to be primitive."[72]

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[70]An interested reader can consult the book by Adelson-Velskii, G., 1983, pp. 59-60, which discusses this aspect in detail.

[71]Adelson-Velskii, G., et all, Machine Plays the Game, Moscow: Nauka,1983. (The last book by the same authors which provided material on positional parameters does not contain this kind of evaluation; and Slate, D., "The Northwestern University Chess Program", Chess Skill in Man and Machine. New York: Springer Verlag, 1977.

[72]Slate,D., and Atkin,L., " The Northwestern University Chess Program", Chess Skill in Man and Machine. New York: Springer-Verlag, 1977, pp. 93-94.

The work quoted above encouraged others to try to determine at least partially (with a large number of parameters assigned intuitive values) the values of a few positional parameters analytically.

> For instance, "Knights are evaluated and king and center tropism and development. Center tropism is a peculiar measure of the closeness of a knight to the center of the board and is computed as 6 minus twice the sum of the rank and file distances from the center of the board to the square the knight is on. The value is then multiplied by 1.6...
>
> King tropism, for knights, is computed differently from that of rooks. We compute 5 minus the sum of the rank and file distances between the knight and enemy king and multiplied the result by 1.2... Like bishops, development of knights means not being on the back rank, the penalty being 9.4 ." (p.97).

Apparently, the weights of a number of positional parameters were not formed analytically: they were based primarily on players' intuition and experience.

Reasons underlying this method of evaluating positional parameters are deep. Subjectivity of judgement is rooted in the indeterminate nature of the game, in the need to adjust the play to the abilities and knowledge of a given player. Analysis of material parameters has revealed that they arose as as result of the rules of the game itself and their weights were calculated in an objective fashion. In general, subjective evaluation of a state is due to the subjective set of chosen positional parameters and their subjective evaluation. These considerations regarding the subjectivity of positional parameters do not rule out the possibility of establishing some averaged out magnitudes based on the analysis of a large number of games by players or machines. Still, these averaged out values would be mere guidelines, i.e they will reveal at first glance parameters which must be taken into account and call attention to those parameters whose subjective evaluations differ substantially from their averaged out norm; reasons for the gaps would have to be explained in order to justify the use of these averaged out values.[73]

M. Botvinnik proposed a totally different approach to positional parameters. Acknowledging the need to evaluate both material and positional parameters, he notes that:

> "Such a method (method by which various averaged out weights of positional parameters are introduced A.K.) of deriving a positional estimate in chess is wrong. A positional factor that yields a positive result in one situation may yield a negative result in another. For instance, are doubled pawns good or bad? The answer depends on the situation. Sometimes doubled pawns are a suitable target for an attack, since one of them cannot support the other. But sometimes doubled pawns contribute to the control of a square through which important communication lines (trajectories of pieces) pass, and then the doubling is extremely useful. The same thing may be said of other factors entering into the positional component of Shannon's prototypical scoring function.
>
> In the chess program under consideration, a quite different decision was made about the positional component of the scoring function, basing it on the control of those squares making up the trajectories that enter the MM. The side controlling the larger number of squares has a positional preponderance."[74] (p.29).

Botvinnik elaborates on this point:

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[73]This question is investigated in detail in the forthcoming book "Indeterministic Economics" with regards to evaluating the state of the firm.

[74]Botvinnik, M., Solution of Inexact Problems. Moscow: Soviet Radio,1979, p. 29.

"Control of fields does not mean control of the whole board, but control of only those fields that may be used in the impending play. Therefore one must strive for control of the field consisting of those trajectories in which the pieces can move, but have not moved yet.

At the node in the search tree where we find ourselves at a given moment, we must unravel all those sheaves of trajectories which have not yet been developed and determine which player has control of the majority of the fields consisting of the trajectories not yet used in the play. This allows us to forecast the result of the play -- the result of a search which, in particular, had to be renounced at the terminal nodes of the variations for lack of resources."[75]

An interested reader can refer to Botvinnik's book quoted above for a more detailed discussion of the methods of assigning weights to positional parameters. I do not intend to compare the two approaches to the evaluation of a position proposed by M. Botvinnik and C. Shannon (or his followers). Again, I refer the reader to G. Adelson-Velskii's book which presents a comparative analysis of Botvinnik's approach.

Finally, evaluation of a state calls for an answer to the following question: "How do we compare the value of material parameters with the value of positional parameters?" The answer given in the literature is very intuitive. Essentially, it claims the the total value of positional parameters should be somewhere between that of a half-pawn to two pawns. Adelson-Velskii notes that the value of the smallest material unit - a half-pawn "must be high enough to exceed the total values of all positional parameters."[76] At the same time, they say "It would be very beneficial to think of positional parameters comparable to the material in value."[77] D. Slate, D. writes "The total of the nonmaterial terms does not usually exceed the value of about a pawn and a half."[78] And finally a quote from M. Botvinnik: "We may assume that a so-called positional sacrifice should not exceed two units of material (this will be made more precise in an experiment)."[79]

H. Berliner notes:

"I differ from some leading practitioners in that I have always been an advocate of positional contributions that can some to more than the value of a Pawn. We have had situations where Hitech considered one side to have almost 3 pawns worth of positional compensation."[80]

This emphasis on increasing the relative weight of positional parameters stems from the experience of professional players. In some cases, they are willing to sacrifice material and not just a pawn but a whole piece in order to improve their position. But here we are jumping ahead of ourselves. I will have a chance to discuss positional sacrifice when I talk about the relationship between combinational and positional styles of play.

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[75]Ibid., p. 38.

[76]Adelson-Velskii, G., et all, Machine Plays the Game, 1983, p. 69.

[77]Ibid.

[78]Slate,D., and Atkin,L., " The Northwestern University Chess Program", Chess Skill in Man and Machine. New York: Springer-Verlag, 1977, p. 95.

[79]Botvinnik, M.M., Computers in Chess: Solving Inexact Search Problems, Springer-Verlag: NewYork, 1984, p. 38.

[80]Berliner, H., "Some Innovations by Hitech", ICCA Journal, Vol. 10, No. 3, 1987, pp. 111-117.

Chapter 4. Class of local problems and stages of the game.

§ 1. Classification of search problems

Having better understood the evaluation of a position we can discuss different types of local search problems. Problem type depends primarily on the structure and the value of the final product associated with each local problem. What determines the constraints of the problem is the current position on the board.[81]

The structure of the final product can be represented in a variety of ways. Our classification is based on four characteristics.

1. How fixed are the values of independent input variables, i.e. some or all of the variables are given as constants or as unknowns; all this is based on the hypothesis of the existence of the true values of the variables incorporated into the objective function.

2. Remoteness (number of moves separating) of the final position from the current one: the final position (relative to a given problem) can be distant or close (even one half-move away) to the current one.

3. The extent to which the structure of the objective function is certain: the qualitative characteristics of its arguments may or may not be well defined.

Table 6. The structure of the objective function of the local problem.

Characteristics of the future positon		Remoteness of the future position from the current one			
		Far		Near	
		Impact on the development		Impact on the development	
The values of the variables determining how fixed is the value of the position	The degree of uncertainty of the structure of the position	strong	weak	strong	weak
Constants	Clear				
	Fuzzy				
Unknown	Clear				
	Fuzzy				

4. The variability of the value of a given parameter as a function of the impact of a current position on the future flow of the game.

Table6 presents all possible combinations of these characteristics in amatrix form.

Table 7 gives names and abbreviations for the 16 types of local problems derived in Table 6.

Table7. Types of local problems and areas of their application

No. Area of application

1. Positional problem with a highly effective procedure for eliminating inferior moves
2. Positional problem; the procedure for eliminating inferior moves is not very developed

[81]Of course, this is only true if we assume that the process is markovian. Actually, it may be worth while in formulating local problems to remember previous moves made in the game. Here, I am going to ignore these considerations.

3. Positional problem given in aggregate variables with a highly effective procedure for eliminating inferior moves
4. Positional problem given in aggregate parameters; the procedure for eliminating inferior moves is not very developed
5. Sublocal problem of the general form with an unexpectedly strong outcome
6. Non-key sublocal problem of the general form (standard algorithm block)
7. Sublocal problem given in aggregate parameters with an unexpectedly strong result
8. Sublocal problem given in aggregate parameters with a weak result
9. Positional goal directed type of problem with sublocal problems
10. Goal directed type of problem with sublocal problems, result weak
11. Goal directed type of problem given in aggregate parameters with sublocal problems; produces drastic changes on the board
12. Goal directed type of problem given in aggregate parameters with sublocal problems; weak results
13. Combinational problem; the amount of material captured is decisive
14. Non-key sublocal problem
15. x
16. Combinational problem; material is won and positional parameters vary.

I want to introduce based on this classification some general definitions pertaining to local problems. If parameters associated with the final position are given (they are known values) this position shall be called a goal.

A problem based on this kind of a goal shall be called a goal-directed problem.

If the number of moves separating the final position from the current one is more than one the problem shall be called protractive; a one-step problem shall be called a condensed problem.

Suppose all the initial as well as final conditions of a goal-directed protractive problem are given. Then the problem reduces to finding an optimal trajectory of moving from one state to the other. This trajectory could, for example, fulfill the requirement of reaching the final state in a minimum amount of time. A function designed to implement this kind of a constraint shall be termed a criterion of optimality. In an extreme case when the choice of the trajectory does not make any difference a local optimization problem reduces to finding just one such trajectory which of course must not violate the rules governing the transformation of one state into another.

Unlike goal-directed local problems local problems that include parameters of the final state whose values are unknown shall be called non-goal-directed problems. Here, a function which evaluates a final position shall be called an weight function. Non-goal-directed problems can be either protractive or condensed.

In setting up a local problem we may have some idea of the value of some part of the final position in question. Our hypothesis does not have to be incorporated into the evaluation of the final position explicitly; it can be represented as a constraint assigned a constant value. These types of constraints represent objective constraints. They should not be confused with global constraints which also stay fixed but which pertain to the local problem in its global form; these constraints are uncontrollable, they are completely determined by previous development.

All the terms associated with the definition of a final position (goal, objective constraints, criterion of optimality, weight function) combine to form the concept of an objective function. It allows us to focus on the general structure of the solution of the local problem in case we are not interested in all its details.

§ 2. Non-goal directed protractive problems.

Let us begin our analysis with non-goal directed type of local problems.

Constraints incorporated into local non-goal directed condensed problems are determined by the current position of the pieces (both one's own and the opponent's). The value of a move is determined by the weight function as the difference between the evaluation of the new position (one half-move) by a given player and his evaluation of opponent's half-move. It is assumed that the opponent responds to your own best half-move with his best half-move. This method of evaluation is based on Zermelo's principle. It states that the value of a move is determined by the maximum value of a position reachable by that move.

All this is rather trivial. It is analogous to a firm trying to maximize its profit or a man trying to maximize the difference between his positive and negative emotions. (It is also akin to the process of setting up local problems in the above mentioned decompositional algorithms of optimization).

In constructing a weight function for a given problem it is assumed that the values of all material and positional parameters included in it are unknown, i.e. all its arguments are variables or unknown quantities. In other words, we cannot predict the exact values of all the parameters determining the value of the position at the end of the search; i.e. we do not know which pieces shall stay on the board, what squares they will occupy, or which positional parameters associated with that particular configuration will come into play.

At the same time, it is assumed that all the coefficients of the arguments, i.e. weights of material and positional parameters are completely determined. The other two components of the objective function cannot be determined outside a given situation. For instance, the "distance" separating the final position from the current one largely depends on the search algorithm being used, the ability to eliminate inferior moves, and the power of the computer. The impact of unknown structure on the future of the game is usually rather weak, i.e it is not meant to accommodate major changes. At the same time, it can only be determined post factum or after the problem has been solved.

If we knew the overall values of parameters included in the local objective function, i.e. values conveying global information about the entire game, then the solution of such a one-step local problem would constitute one segment of the optimal trajectory representing the solution of the global problem. But as I have mentioned many times before such global evaluations cannot be deduced within an algorithm of an inexact game: we have to settle for partial evaluations. Therefore the value of the local objective function is not exact. This has lead many people involved in the development of chess algorithms to expand the search horizon, i.e. increase the depth of the search. It is clear that the lesser the depth of the search the greater the probability, all other conditions being equal, of undiscovered flaws in the positions beyond the horizon: evaluation of a position at a given step serves merely as a guideline for future play.

Now, since the values of the variables comprising the objective function are not known, the local problem at the start of the procedure would, in theory, involve just one move - two half-moves. The actual number of moves looked at, i.e. the horizon of the protractive problem depends on several aspects including the power of the algorithm and the machine being used. It was precisely this uncertainty over the length of the sequence of local one-move problems that stimulated me to call this sequence of local problems a protractive local non-goal directed problem.

As I have noted in the section on general description of a chess algorithm, the number of possible moves in the game is enormous so the machine is really limited in its ability to exhaust all the variations. Therefore, the solution involves a well-known trade off between the depth of search or the horizon and the width of search or the number of moves examined at each step.

It was precisely this kind of approach to the formulation of a local problem and a corresponding objective function (viewed within the global framework of the entire algorithm) that was suggested by Shannon, C., 1950. This approach, somewhat modified, is still being used today by most algorithms of the game.

At the same time, there are strong arguments against Shannon's approach. M. Botvinnik thinks that

> "The first method, however, is hopeless as means of finding a good solution to an inexact task, as we shall show by applying it to the game of chess. On the average, each side in a chess game has 20 available moves. Suppose we start in a position where White is to move; all 20 moves of White's pieces must be included in the search tree. Then it is Black's turn; for each of White's 20 moves, Black has 20 answers, so that the tree contains 400 Black moves. If we extend the variation to include another move by White, the tree will complete moves -- six half-moves ("plies") -- it will contain some 67,000,000 nodes! The size of the tree is an exponential function of the depth.
>
> If we develop a suitable algorithm for a chess program, the resources of the computer (memory and speed) are less significant than might be supposed at first glance. Whenever we extend the depth of the variation by a move, the resources of the machine must increase by several factors of 10; this is impossible in practice. Thus the main element is not the power of the computer, but the skill of the programmer, i.e., the extent to which he has contrived to depart from the fundamental principle (a full-width search) in order to economize on machine resources."[82]

I agree with Botvinnik's criticism of protractive problems but I do not think they should be ignored altogether. Not only is the protractive approach capable of competing with other approaches in formulating local problems, but it can also be used in conjunction with other techniques. I shall elaborate on this idea further on in the chapter.

§ 3. Goal-directed protractive problems

In the present section, rather than concern myself with the ways of improving the set up and the solution of non-goal directed protractive problems, I shall try to apply other

[82] Botvinnik, M.M., Computers in Chess: Solving Inexact Search Problems, Springer-Verlag: NewYork, 1984, pp. 2-3.

principles to resolve these problems, principles rooted in the goal-oriented approach especially its protractive variety.

In any goal-directed problem it is assumed that the goal can be reached without breaking the rules of the game. A reader may be surprised by this seemingly naive and indispensable requirement. But there is a branch of chess theory called retrograde chess.[83] There are positions that are legal as far as the positions of the pieces are concerned but which can never be reached from the starting position if we follow the rules of the game. The purpose of retrograde analysis is to prove that there exists no backward sequence of legal moves from a given position (legal in itself) that would eventually lead back to the starting position.

Therefore, when we form a long-term goal we have to be careful to avoid "retrograde goal".

Assume there is no risk of such a goal being set (it does not occur frequently anyway so we can ignore it for the time being). Still we have to set a goal that is realistic, not just wishful thinking on our part (that is random in the extreme case). So far, there is no formal procedure to accomplish this task. I shall attempt to show below that the solution to this problem calls for a rather sophisticated analysis of the structure of the state (current position) - uncovering positional features peculiar to it. This would ensure that the goal set is more or less reasonable.

We shall begin our analysis with attractor type of problems. Its impact on the outcome of the game is strong and its specific goal is given over a limited set of material parameters (problem number 13). A typical goal for this kind of problem is to try to capture opponent's material which would virtually defeat his position.

In other words, positional considerations can be ignored when the goal is to capture a lot of enemy material. The criterion of the local problem reduces to finding an optimal trajectory linking the current position with the final state to be attained.

It is customary to call this kind of problem combinational.

The section entitled "Forced play and how to prepare for it" of Adelson-Velskii's book[84] quoted many times presents some interesting examples of a goal (only in simple cases like mate in one) which consists of capturing enemy material without searching through all the variations branching from the current position.

It is easy to see that combinational problems have some very profound advantages: its solution gives one side an upper hand by generating a swift attack aimed at winning enemy material.

Nevertheless these advantages are rather conditional, they are realizable only under some very special circumstances. Although the goal associated with a combinational type problem is relatively clear, it suffers from a number of drawbacks which stem from the need to determine the values of different pieces. Of course, if one side is up a heavy piece then the evaluation of a position is generally not a problem. Here, "generally" excludes,

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[83]Smullyan, R., The Chess Mysteries of Sherlock Holmes. Hutchinson, 1980.
This branch of chess, sometimes called "chess logic" is also becoming part of computer chess. See Alden, B. and Bramer, M., "Development of a Problem for Solving Retrograde Analysis Chess Problems", Advances in Computer Chess, 3, ed. by M. R. B. Clarke, Oxford: Pergamon Press, 1982, pp. 121-137.

[84]Adelson-Velskii, G., et all, Machine Plays the Game, Moscow: Nauka, 1983, pp. 84-85.

among other things, an intentional sacrifice on the part of the opponent. (The subject of a sacrifice in chess will be addressed in the next chapter.)

But what happens if the consequences of a combination are unclear or if we cannot come up with such a combination at all?

> "Maybe a combination does not exist at all, may be he [player] is just chasing a ghost? A search for a combination may turn out to be futile. To base one's play on the existence of a combination is a very dangerous enterprise."[85]

How then is the game to be played? It must be played positionally. Positional play does not preclude combinations. On the contrary, it creates predisposition towards a combination. To quote Lasker once again

> "Hence, he [Steinitz] concluded that some characteristic, a quality of a given position must exist that to a discerning eye would indicate the success or the failure of the search before it was actually undertaken. And this characteristic, if explicable by reason, of what could it possibly consist if not an advantage or a disadvantage...And an advantage, if reasonable, what could that be except the thing that was generally termed so: greater material force, greater mobility, greater effect against the King..."[86]

So positional play makes sense if it eventually leads to a combination. To rely exclusively on combinations is flawed for it rings of primitive pragmatism. Purely positional style has the disadvantage of being over speculative. The fusion of these two styles, positional play being confirmed by a crowing combination, breeds very effective results.[87]

To begin with, we distinguish two types of goal-directed positional problems: tactical and strategic. The first can be described in the following way: a goal is given over a limited set of material and positional parameters, it is remote, rather fuzzy and has a strong impact on the future (i.e., problem number 11). Preparing an attack against the opponent's king serves as an example of this kind of problem. A sequence of moves linking the goal with the current state is not only long but also fuzzy which forces us to set sublocal problems, i.e. intermediate local problems. Unlike combinational problems, intermediate local problems aim not at gaining material advantage but at improving current position by strengthening positional parameters. Sublocal problems can in turn be classified as key or non-key problems.

Using our previous classification of objective functions, the primary characteristic of a key sublocal problem is its strong impact on the rest of the game. Its other feature is that the objective function corresponding to a position which is not too remote from the current one is represented in the following form: positions of one or several "mobile" pieces is set as a goal. These pieces determine the feasibility of a successful attack against enemy king; the rest of the pieces remain stationed at their previous positions.

A famous game between T. A. Romanovskij and V. D. Ragozin played in the Second International Tournament held in Moscow in 1935 is an example of this kind of a problem.[88] Romanovskij asked Capablanca to comment on the game after it had reached a certain

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[85] Lasker, E., Lasker's Chess Manual, Moscow: OGIZ, Fizkultura i Turizm, 1937, p. 251.

[86] Lasker, E., Lasker's Manual of Chess, Philadelphia: Mc Kay Co, 1947, p.197.

[87] This situation echoes the development of science which represents positional play and engineering which is combinational. Their evolution and mutual interaction has lead to the greatest advancement of scientific-technological progress.

[88] See Adelson-Velskii, G., et all, Machine Plays the Game, Moscow: Nauka, 1983.

stage in the middle-game. Capablanca answered that the game can be won in "three moves". By a move he meant a key sublocal problem having its own objective function based on a number of constants representing positional parameters, a problem whose final goal is not too remote from a given position, and a problem solution of which has a strong impact on the future flow of the game (13 problem).

The objective of non-key sublocal problems is to improve one's position by "consciously" strengthening positional parameters. These problems are well defined and are evolutionary in nature (problem number 14).

But suppose we are unable to formulate a strategic positional problem. As E. Lasker wrote in his chess manual, when playing the positional style of chess it is extremely important to try to find out "what plan to follow in a balanced position when nothing provides a clear target of an attack or a defense?"[89]

Lasker points out that in this case it is necessary to accumulate small positional advantages as recommended by Steinitz. In other words, we have to set up local problems similar to the ones mentioned above - non-key sublocal goal-directed problems.

Note that in the event this kind of a problem defies formulation we have to switch to a protractive type of local problem.

§ 4. Com-positional problems

Two types of search problems looked at so far - goal and non-goal directed ones should not really be opposed to each other. In fact, they can interact in many interesting ways. One special case is a combinational type of a goal directed problem being used for material parameters and a non-goal directed problem for positional parameters. It was precisely this approach to local search problems that was proposed by M. Botvinnik.

> "In chess the goal of the inexact game is to win material. A similar goal may be found in an arbitrary game that models a control system, and in an arbitrary inexact task. To attempt to solve an inexact problem without having formulated the goal of the corresponding inexact game is to waste time. This goal is the basis of a strong algorithm for the solution of an inexact problem, and the basis for development of a deep and narrow tree. We shall see later why this is so.
>
> The goal of a game says what our aim is; only when we know this can we identify courses of action that cannot lead to our target, and exclude them from the tree. Knowing the goal lets us define the lines along which the search is to occur.
>
> The goal lets us direct the search; the scoring function lets us evaluate and stop a variation. The goal lets us form a search tree; the scoring function lets us strike a balance.
>
> The scoring function acts together with the goal of an inexact game and is therefore itself inexact. As distinct from the goal, which must be unique, the scoring function consists of two components: the first component allows us to evaluate the results obtained with the limits of the truncated tree; the second forecasts the possibility of reaching the goal beyond those limits.

[89]Lasker, E., Lasker's Chess Manual, Moscow: OGIZ, Fizkultura i Turizm, 1937, p.223.

The first component yields an exact answer (with the limited precision already determined) to the questions concerning the goals that have been reached; the second (the positional estimate)[90]

gives a preliminary answer to the question of what will happen later, when the boundary of the truncation has moved further away. Taken together, these two components determine the value of a concluded variation."[91]

It seems to me that the com-positional type of problems is also typical of human behavior. Reducing them to a combinational type of problem represents an attempt to simplify one's behavior. Here, I would like to make another digression to discuss the issue.

Digression 2

It was during the sixties that my wife, my two sons - 15 and 8 years old, and I were coming back to Moscow after a vacation. We had to take a night train and the ride was about 12 hours long. Naturally, we wanted first class tickets. This way we would get private compartment with beds. In addition, tickets are not as expensive as they are for luxury class seats. But it so happened that we did not reserve our return trip tickets ahead of time; we did not really plan to go to that vacation spot, it turned out to be much less than perfect and so we decided to come back home and spend the rest of our vacation there.

When we arrived at the station I went over to buy tickets. The cashier told me that the station superintendent is in charge of selling first class tickets. The superintendent turned out to be a rather nice lady. After hearing my story she told me that she had only three tickets available in the sleeping car. One ticket would have to be in the regular car. I agreed. She gave me a permission to buy the tickets so I thanked her and went over and bought the tickets. I was touched by the woman's courtesy and wanted to to thank her. So I walked over to the station cafeteria to buy her some chocolates. As usual, there was a long line of people waiting to be served. Luckily, my two sons were there already. When they asked me what I was doing there I told them my intentions. They were quite shocked. "It is dumb, they said, to spend money on chocolates for some woman you don't really know who is really not expecting any gifts, whom we shall never see again and who could not do anything else for us anyway."

Well, I did not succumb to their rational arguments. I went ahead and bought the chocolates and gave them to the woman. She was very touched since she was not expecting any gifts at all. She asked me to wait a minute and left. When she returned she told me that she managed to find four tickets in one car. I thanked her sincerely and walked over to the ticket window to exchange the tickets. My children accompanied me. After looking at my old tickets, the cashier told me that I made a mistake by buying all adult tickets. She gave me new tickets plus some money back. The amount was much bigger than the price of the chocolates. My children were startled to see courtesy rewarded so quickly.

What is the point of the story? It seems I was solving a combinational problem having a clearly formulated goal: to get sleeping car tickets. The rules for acquiring the

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[90]According to Botvinnik, M., Solution of Inexact Problems, Moscow: Soviet Radio, 1979, positional value represents a ratio K_w/K_b where K_w and K_b is the number of squares in the trajectory controlled by white and black respectively. (p. 144).

[91]Botvinnik, M., Computers in Chess: Solving Inexact Search Problems, Springer-Verlag: New York, 1984, pp. 16-17.

tickets were well defined: cashier - superintendent - cashier. The criterion of optimality was very clear: to get the tickets in a minimum amount of time. The problem could be solved by going through all these steps with well-defined material goals and choosing a ticket window with the shortest line. This is how my children were thinking, at least on the semi-intuitive level.

As for me, I took both relational parameters and material expenditures required for their realization into account. The weight of these parameters was incorporated into the final state along with the values of such material parameters as tickets. Here, the relational parameter of special importance was mutuality; it was her emotional reaction as such (actually, this applies to all positional parameters) that was important to me independently of the final material result.

Since the weights of positional parameters and material expenditures at the final and intermediate stages were unknown to me, the problem of ticket acquisition (that stage of it anyway) transformed into a non-goal directed problem with the values of parameters unknown.

The problem acquired com-positional nature. Weights assigned to positional parameters as well as to material expenditures were based on my personality. Another person with a different personality and a different system of values might have acted differently under the same circumstances and his behavior would have also been totally justified. There is no reason to think that everyone must assign importance to such parameter as mutuality. The reward received for this kind of behavior was largely a result of a series of happy coincidences (the lady in charge happened to have some extra tickets, she was sensitive to another person's thoughtfulness, etc). It does not follow that the reward had to come immediately. In fact, it could have never come at all! Do not discussions concerning righteous behavior revolve around the same issue? I think that the existence of individuals with different systems of values is very important for development. Therefore, acting under the conditions of uncertainty and "trying to be yourself" (trying to preserve one's individuality) represents a "continuous" growth of potential of the individual in question. Variety makes total potential of people as a whole grow as well.

Now, what about the solution of the problem? It reduces to some well known methods of solving similar problems under the conditions of uncertainty. Solution proceeds on contingency planning basis: at each step we try to choose the best possible course of action based on the current situation.

§ 5. Stages of the game

Our next topic deals with the stages of the game. Each stage can be viewed as a macrolocal problem(just as in biology, a certain set of molecules produces a micelle). Once again, we follow the systems approach in selecting the criterion for stage classification.

We shall consider the functional aspect of the problem first; this approach is player rather than machine oriented.

A well known grandmaster D. Bronstein proposed a very interesting classification of game stages based on the functional approach. These are the stages of the game as perceived by each side: 1) creation of a shelter for the king - a well protected command post; it is neither a castle nor is it a pill box since "a king is not going to fire any shells from this fortress of his "; 2) formation of pawn chains - the skeleton of the position; 3) "allocation" of the pieces, one's mobile resources according to their "personalities" and the intentions of the player; 4) crossing the "equator" for the purpose of invading enemy territory and destroying or weakening his position; 5) "active defense" is when an opponent attacks a given player and it becomes necessary to weaken the former to a degree when "he admits his defeat after the smoke has cleared "; 6) attack against the enemy king - this involves breaking through the pawn chains, capturing space, attacking weak pawns and advancing one's own pawns with support from the pieces.

D. Bronstein proposed a number of techniques to help fulfill the above mentioned functions. For instance, "creation of a shelter for the king" can be implemented not just by placing a king near the edge of the board (as we know, at the start of the game the king is placed in the center). Bronstein recommends not just building a "shelter, but a castle fortified with double walls made of pawns. A king should be provided with personal bodyguards - a knight, a bishop, a rook, and four pawns."[92]

Bronstein proposes to build such a structure by means of a series of moves which includes castling. (p. 18).

Bronstein's approach seems original but lack of literature discussing his ideas makes it impossible for me, an amateur in the game, to judge the validity of his approach.

I want to focus on a more traditional method of stage classification. It is based on the structural features of a position. This approach does not ignore the functional aspect of the game but it emphasize structural characteristics. I think it makes sense to combine the functional and the structural approaches in order to gain deeper understanding of the overall problem.

In my opinion the structural approach incorporates two parameters: total number of pieces on the board and their mobility (which reflects both the position of one's own pieces as well as the opponent's). Here, it is assumed that each of these parameters is represented as a dichotomy, i.e. it can only assume two values. Matrix-table 8 shows all possible combinations of these two structural dimensions and their respective values.

Table 8. Mobility and number of pieces

Mobility	Number of pieces	
	Large	Small
Mobile	middle game	final endgame
Immobile	opening	early endgame

These four combinations correspond to traditional classification of the stages of the game, i.e. the opening, the middle game and the endgame;[93] "early endgame" stage owes

[92]Bronstein, D., Teach Yourself the Game, Moscow: Fizcultura i Sport, 1981, p. 17.

[93]Simplifying things a bit, we can say that human behavior strives to maximize the difference between positive and negative feelings.

Individual choose as to the course of action in a given environment is limited by his intellectual and emotional abilities. Using chess as an analogy, we can mention several different mechanisms used by the people of different ages in making a decision. These mechanisms are rather specific in the childhood stage and during the old age which is analogous to the its inclusion to the fact that the accepted meaning of the term denotes "final endgame" stage.

Here, I am going to skip over the processional aspect of classification of game stages.

In Table 9, I made an attempt to combine the structural and functional aspects of various stages with its process counterpart. Some ideas in this table are borrowed from the literature and some are my own.

Table 9. A multi-dimensional systems approach to the classification of the stages of a chess game

Stage of the game	approach underlying the definitions		
	functional	structural	process
Opening	1. Development of the pieces* 2. Mobilization of the pieces**	1. Limited mobility for a large number of pieces	1. Deep analysis of a large number of variations; knowing them allows both players to maintain equilibrium up to a certain point.
Middle game	Trying to achieve positional superiority***	1. A large number of mobile pieces 2. Passive kings****	Determining the best combination between the depth and the width of the search to ensure an optimal solution
Endgame	The goal can be set up as a terminal position, i.e. a mating position (or clearly won position)	1. A small number of mobile pieces**** 2. Kings are active in the struggle *****	A position whose outcome - a win or draw, can be verified by an analysis with both sides playing their best moves******

*Lasker, E., Lasker's Chess Manual, Moscow: OGIZ, Fizkultura i Turizm,1937.
**Capablanca, H. R., Chess Manual, Moscow: Fizkultura i Sport, 1975, p.31.
***Ibid, p. 36.
****Ibid, p. 42.
*****Lisitzyn, G., The Closing Stage of the Game, Leningrad: Lenizdat, 1956, p. 5.
******Portish, L., 600 Endings, Moscow: Fizkultura i Sport, 1979, p.6

Although the above table is largely based on the structural features of a position, its multi-lateral approach puts the peculiarities of each stage in relief. For example, the structural connection between a large number of rather immobile pieces and a method of play based primarily on prior experience (rather than optimization or self-learning) allows us

opening and the endgame. It seems that the years between these two stages involve the use of more complex mechanisms. Here, we probably have some substages with their own peculiar features.

to define the opening stage of the game more precisely. Structural characteristics of this stage could, as such, apply to other stages as well. There exist middle- game positions having a large number of rather immobile pieces. The connection between the structure and the process based on previous experience links these characteristics with the function of piece mobilization thus giving a holistic definition of the opening stage.

We should also note that the structural aspect of the game does not determine either the function or the process: it merely brings greater degree of correlation between them.

The endgame stage is characterized by a small number of pieces on the board. This obviously simplifies the task of predicting the "final position" since the number of possible variations is not so large.

A final position, i.e a position which leads to an immediate mate shall be termed terminal position.[94] The presence of terminal positions allows for the implementation of backtracking algorithms which start at the final position and work back to the initial ones. Here, the actual terminal position may have a direct impact on the criterion associated with each intermediate position. Since the number of terminal positions in chess is enormous, an algorithm is basically forced to move "forward" from the current to the final (relative to a given task) position. This gives rise to a very difficult problem of formulating a local criterion to evaluate intermediate positions.

These features of endgame stage reveal why this stage can be handled by an optimization algorithm[95] which ensures that a terminal position is reached in a minimum number of moves.[96]

Nevertheless, terminal positions can be set up even at the beginning of the game when there is a large number of immobile pieces. Example of this is the so called "Fool's mate": white to move and mate in three. In reality these considerations apply only to the beginners. At more advanced levels of play, terminal positions are so remote from the initial one that they defy clear-cut formulation not only in the opening but also in the middlegame.

So, we have looked at the three stages of the game from the systems point of view. I recommend that the reader consult literature on this subject to fully appreciate the game (a partial list is given in references to Table 7). These books contain very interesting ideas on the various stages of the game. Unfortunately, these ideas are not presented systematically: the role of each aspect in the analysis is not stressed; its relation with other aspects of the game is not incorporated into the definitions of various stages. To illustrate this point, I want to quote from C. Shannon, the founder of computer chess. The quote contains a very concise description of the three stages of the game.

> "A game of chess can be divided into three phases, the opening, the middle game, and the end game. different principles of play apply in the different phases. In the opening, which generally lasts for about ten moves, development of the pieces to good

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[94]Terminal positions can occur at an earlier stage - a state which leads inevitably to a mating attack. This feature is incorporated into some computer chess programs which warn the opponent of a mate in a certain number of moves.

[95]"...while in the middlegame players' plans are largely determined by their tastes and fantasy, in the endgame they are determined bu the position in question. Everyone must more or less follow the same plan which is typical for a given position independently of his style or tastes. In fact, many endgame positions are nothing else than logical problems sometimes having a unique solution." (Averbasch, U., Things to Know About the Endgame, Moscow,1979).

[96]This may affect the "fifty move" rule which states that if no exchange of pieces takes place or no pawns are advanced then one side can claim a draw.

positions is the main objective. During the middle game tactics and combinations are predominant. This phase lasts until most of the pieces are exchanged, leaving only kings, pawns and perhaps one or two pieces on each side. The end game is mainly concerned with pawn promotion. Exact timing and such possibilities as "Zuzwang", stalemate, etc. become important."[97]

Finally, concerning the level of development of different stages of the game.

"Progress in present day general theory [of chess - A. K.] has been primarily in the opening stage. The ever growing amount of chess literature, "the information boom", has lead to the theory of opening stage "outrunning" other stages of the game. Study of openings is being conducted on all fronts, both in depth and in width. There appeared a breed of fanatics theoreticians who like moles dig deeper and deeper occasionally reaching beyond the middlegame and into the endgame.

Evolution of opening theory has resulted in "real" game starting somewhere around move 20 or even 30. All previous moves are well known in advance. In these sometimes very sharp situations the decisive factor is frequently the player's book knowledge rather than his abilities.

...We should note that at the present time the middle game is the most underdeveloped area of chess theory. Numerous positions classified one way or another are of little help when it comes to real theory of the middle game, a general high quality theory which would really help orient practical players in the immense ocean of the middle game."[98]

[97]Shannon,G., "Programming a Computer for Playing Chess", The Philosophical Magazine, vol.XLI, 1950, p. 265.

[98]Averbasch, I., "Chess Today", in a book by Averbasch, I. and Taimanov, M., World Championship Match Karpov vs. Kasparov. Moscow: Fizcultura i Sport,1986.

Chapter 5. Styles of the game

§1. The subjective and the objective in chess: aesthetic and scientific methods.

Chess practise has shown that one can play only so well just by studying and playing the game. Further progress requires creative abilities. These abilities can be developed if a predisposition toward them is there. Creative abilities are crucial in chess. A youngster who lacks practical experience but possesses creative abilities can achieve very impressive results (the situation is similar in mathematics, sports, art but not in physics, social sciences, history, etc.). If a player does have creative abilities it would be interesting to see them at work, to see how they relate to the various methods of play, especially the combinational and the positional methods. Here, I want to discuss one aspect of this problem which in my opinion is of general methodological significance.

Combinational style of chess requires that a player have the talent to discover a combination, and not only its ultimate outcome but also a program of its realization. Therefore, having discovered a combination and having put it in more or less precise terms a player can verify it (sometimes with his assistants) before playing it in the actual game. Checking a combination for holes can produce absolutely unambiguous results, at least in the probabilistic sense of the word.

Positional play is a totally different story, especially at its non-goal directed stage or if it revolves around the accumulation of small advantages. First of all, a player can improve his position by introducing new positional parameters together with his individual scale of values based on his experience, knowledge, and abilities. A player possessing these abilities is in the same situation as a combinational player.[99]

Moreover, player's subjective experience in playing certain type of positions is not the only factor which determines his evaluation of a given position. His judgment also depends on his experience in realizing various positions, i.e. at transforming them into a combination. But what is the right time for this transformation? At what point of the game does a player, given his experience and abilities, start to look for potential combinations?[100] Here, the subjective factors prevail. One factor is player's confidence that given a certain position he will be able to squeeze most out of it. Under these circumstances, a positional player will be unable to demonstrate to his assistants a clear-cut way to realize his positional advantage: it is subject to his expertise, creative abilities, and originality.

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[99]"In the end, all my valuations originate from my experiences: my first losses and wins which gave me pain and joy; my first draws that called forth in me a variety of sentiments; my first analysis, which was crude and faulty. From then on I valued and continue to value; and with practice I became capable of more exact valuation. And from this rough material is generated, by continued trial and intelligent criticism, the series of valuations by which the master arrives at his conclusions." "These valuations, and this is Steinitz's most important idea are supposed to guide us. They are like a compass for the sailor swimming in an ocean of combinations." (Lasker's Manual of Chess, Philadelphia: Mc Kay Co. 1947, p.191 and Lasker, E., Lasker's Chess Manual, Moscow: OGIZ, Fizkultura i Turizm, 1937, p.194).

[100]M. Tal advances some interesting considerations in his account of Karpov-Rashkovskii game: "Karpov has played very consistently throughout the game but being strategically inclined towards king's Indian defense he had overlooked the opportunity to end the game thirty moves earlier.
...This experience came in handy. The third game of the Karpov-Spassky match player in half a year witnessed the same opening set up for black chosen by the former champion. Karpov won the game convincingly."
Tal, M., "All is ahead". Introduction to A. Karpov's book Selected Games 1969-1977. Moscow: Fizcultura i Sport,1978.

Therefore, positional judgements can be controversial while a combination - its end result and its soundness can be judged by the experts unequivocally.

Subjectivity and objectivity in the styles of play correspond to scientific and aesthetic methods respectively. Which of them dominates depends on the stage of development. Let me clarify this statement.

It seems to me that the notion of the value of a position is nothing else than an aesthetic evaluation of its beauty. The word beauty does not apply to the end result of a combination since victory may be close at hand due to big material advantage. Actually, there are two factors determining the beauty of a position: on the one hand, a high value assigned to a given position merely signifies its greater predisposition towards future development. On the other hand, position's evaluation is not sufficient for the program to link constructively the material available on the board with the outcome of the game.[101]

So far, we have examined the combinational and the positional styles of play from the standpoint of the end result. Now, let us view them from a different perspective - the methods used in each stage to achieve the goal. What delights us in a combination is the unexpectedness of the moves. It is the great weight assigned to such parameter as "deep understanding" and the drastic departure from the orthodox and the stereotypical that induces in us this feeling of beauty of the combination. We can also be certain of the soundness of the combination initiated by the player's creative genius for it can be checked carefully and unambiguously. Positional style of play geared towards achieving a good position frequently does not impress us with its paradoxical moves. Positional moves, especially in non-goal directed problems or goal directed problems centered around the accumulation of small advantages, do not go against the grain of our common sense, especially that of professionals for whom positional style has become second nature, their traditional mode of thinking.

So, in positional chess beauty manifests itself primarily in the structure of the position. Its unorthodox nature represents one of the preconditions for future development. Beauty of a combinational style lies in the method of play which departs from stereotypical understanding.

Now let us ask ourselves the following question: can the category of beauty used in evaluating a position be applied to the combinational style of play? One of the most vulnerable aspects of a combinational style is formulating a goal. Goal formation which makes combinational style so attractive is at the same time the most obscure phenomenon: we frequently do not know what motivates players in their choice of a particular goal. It seems that target selection (setting a goal) is directly related to finding a "weak link" in the position of your opponent. Could it be that the weak link is less beautiful or even ugly? Beauty is not an absolute - it has gradations. A system as a whole can be beautiful and yet have some ugly parts. They detract from the beauty of the whole and their "development" can be very detrimental to the system as a whole. Therefore, the initial step in our search

[101] I think that the concept of beauty in chess as seen by A. Roizman is quite similar to my own ideas on this subject. "Beauty in chess is really a multi-dimensional concept. Experts will speak with understanding of the beauty of say a positional maneuver or endgame technique. But nothing can really compete with the beauty of a combination. They are unexpected, effective, and seem very paradoxical. At the same time, they are very logical at some deeper level. All this makes a strong impression on the lovers of this ancient game. Another important factor is that combinations, because they are forced, are accessible to ordinary players." Roizman, A. J., Chess Miniatures, Minsk: Polymia, 1978, p. 3.

for a combination should be analysis of a position typical of positional style in order to pinpoint the least beautiful or perhaps even ugly spots in the opponent's position. For instance, one can start by ranking opponent's pieces based on their unconditional values supplemented with positional characteristics associated with each piece. Does not this total sum reflect the degree of beauty? And does not its negative value resulting from the prevalence of negative positional parameters indicate the extent of ugliness?

Of course, ranking opponent's material is only the first step. A player then faces a very non-trivial task of picking a specific target based on the current position on the board, i.e. he has to weight the benefits associated with achieving a particular goal against the expenditures for its realization. Is not this how chess players think on the intuitive level when they select a target? Do not animals behave in the same manner when hunting for prey?[102]

§ 2. A sacrifice in chess

The difference between the subjective and the objective in chess becomes very vivid when we look at a sacrifice, especially when we compare positional sacrifice, the most difficult one to judge, with a combinational sacrifice.

History of chess provides us with a unique opportunity to follow the evolution of the idea of a sacrifice, the interplay between its costs and benefits and its role in the most general case of creation of a position. It seems that chess players go through at least four stages in their understanding of the concept of a sacrifice. A beginner strives to gain at each step of the way, he never makes a sacrifice.[103]

The next level of play involves the idea of a "pseudo" combinational sacrifice.[104] Here, a player is willing to make a sacrifice if the entire chain of moves of the combination which results in material gains (or an overall victory) is 100% certain.

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[102]It seems to me that something similar takes place when the category of beauty is used in mathematics. In proving theorems, mathematicians often face the same sort of problems as do chess players: an exact algorithm is not available so we are forced to prove the hypothesis from the beginning and not from the end. The tree of "possible moves" grows very quickly preventing us from linking the current state with the end result. Therefore, we must be able to evaluate a position - an intermediate stage, a stepping stone for future development. It is reasonable to think many good mathematicians are guided by the criterion of beauty. But this is a rather trivial statement. What is less trivial is employing the notion of beauty to check if a proof is really sound.

Looking at the relationship between human patterns of thought and computer idols, a professor from Columbia University B. G. Moishezon noticed this very interesting phenomenon.

"...every professional mathematician whose work always entails finding mistakes knows that using a purely logical approach, i.e. checking each step one a time, does not lead to success. Mathematicians discover mistakes when they step aside or go back, i.e. when they use informal methods, when they compare the results to be checked against other information available to them. Only by comparing, deliberating, and going back and forth can a mathematician find the mistakes." Moishezon, B. G., "On Human Thinking and Computer Idols", Vremia i My, N°86, 1985, p. 82.

I agree with B. Moishezon's statement. I just want to make one method for uncovering mistakes in mathematical theorems more concrete.

At first glance, it seems that to prove the soundness of a theorem we just have to check each step for logic. Why then is it so difficult to pinpoint mistakes in the proofs? Why must decades sometimes pass before a logical mistake in the proof is discovered? What happens is that our mind is unable to check each little step for all details, especially its connection with parallel results achieved a few steps earlier. Thus, we are forced to approach a proof positionally. In proofs saturated with statements (material) and their interrelations (positional parameters) we must seek out those which are potentially liable to lead us in the wrong direction. Here, the primary candidates for a possible mistake are the ugly parts of the proof.

[103]This kind of immaturity in chess is typical of the behavior of some adults too. They want immediate results and are thus unwilling to make sacrifice today for a gain in the future.

[104]Spielman, R., Theory of Sacrifice, Moscow: Fizcultura i Turizm, 1936.

At the next stage of the learning process the idea of a combinational sacrifice is supplemented by a probabilistic evaluation of the outcome. Naturally, carrying out such a combination requires more skill on the part of the player.

Finally, by the time a player has reached the fourth stage he is willing to make a sacrifice for the sake of improving his position. This kind of a sacrifice is called positional sacrifice.

Lasker, E., 1937 defines positional sacrifice in the following way:

> "Nowadays, a sacrifice based on the evaluation of position is called a "positional sacrifice". For instance, a pawn is sacrificed for the sake of development, a piece is given up for two pawns and an attack, and a queen is sacrificed for a rook and a bishop - all this for the sake of "position". These are not combinational sacrifices for the network of possible variations is too large, but they are surely the forerunners of combinations. And always is a positional sacrifice a sign of character and ability to think independently." (p.235).[105]

Positional sacrifice as well as positional style in general is a more recent development than a combinational style of play. Positional sacrifice does not preclude a combination, in fact they complement each other. Sometimes, when the possibilities of a combinational sacrifice have exhausted themselves it transforms into a positional sacrifice. This is what happened to king's gambit as played by white. Falkbeer's countergambit and other active modes of play for black have lessened the popularity of king's gambit.

> "It is presently played in the positional style, without ambitious attacks and sacrifices typical of the past".[106]

In making a positional sacrifice a player does not know exactly how the game will develop. The greater is our ability to foresee the consequences of a positional sacrifice (more precisely, to see the advantages of making a sacrifice) the closer it is to a combination.

So, a player is really uncertain of how and when a positional sacrifice will justify itself. In fact, a player making a sacrifice must be confident that his knowledge and talent is sufficient to realize his positional advantage at some future point of the game.

> "A player who decides to make a positional sacrifice bears additional responsibility: his play will have to be very forceful and precise. "Second rank" moves that are easier to find and that often turn out o'kay in normal positions will probably not do. They might lead to a loss of positional advantage and the initiative while the material advantage still belongs to the opponent."[107]

All this means that one must play at about master's level before he can afford to make a big material sacrifice for the sake of positional advantage. A combinational sacrifice determines the course of the game (completely or partially); an advantage gained from a

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[105]An Austrian grandmaster R. Spielman also gave a definition of a positional sacrifice:
"A chance to convert power into material and material into power is a wonderful feature of the game, may be this is their greatest secret. "
Commenting on this quote from Spielman, P. Kondratiev writes that: "Subjecting myself to the dangers of analogies and metaphors, I should say a positional sacrifice exemplifies the law of the conservation of chess energy." Kondratiev, P. E., Positional Sacrifice, Moscow: Fizcultura i sport, 1983, pp. 3-4.
Much of our further discussion of positional sacrifice is based on Kondratiev's book, 1983.

[106]Roizman, A. J, Chess Miniatures, Minsk, Polymia, 1978, p.5.

[107]Kondratiev, P. E., Positional Sacrifice, Moscow: Fizcultura i sport, 1983, p. 4.

positional sacrifice must be realized under the conditions of uncertainty. This points to the fact that a positional sacrifice is accessible only to players with certain innate abilities. Players differ not only in the amount of chess knowledge they had received or who their teachers were but also in the methods of play which, given our current level of knowledge and pedagogical skills in presenting the material, cannot be assimilated by everyone who wishes to use them (here, I do not mean to rule out the possibility that as our knowledge becomes deeper and teaching methods improve anybody can be taught to do anything).

In digression 3 I want to use an example of human patterns of behavior to illustrate the concept of a sacrifice in chess. For me the value of a particular point of view is revealed by whether or not it leads to conclusions opposite in sign to those arrived at on the basis of traditional methods used in that field, i.e. every day common sense.

Digression 3.

The notion of a positional or a combinational sacrifice frequently comes into play when a person is faced with having to make a decision .

I remember a conversation I had with a recent immigrant from the Soviet Union. A gifted engineer with the knowledge of English he received a decent job and good salary upon coming to the United States. After some years past his son was accepted to a private university. It was hard to pay for the son's education since much of the salary was spent on everyday things and their savings were negligible. This immigrant family had relatives in the U.S.: two older childless women who belonged to the lower upper class. (They were able to maintain their previous standard of living by living of interest on the interest gotten from their savings). These women maintained a rather close relationship with the immigrant family. Yet they offered no money to help finance the boy's education. So, this engineer complained to me about the stupidity of his relatives. He thought that by giving his family the financial help that they needed his relatives could have greatly improved their relations with his family and at the same time not suffer too much financially.

I disagreed telling him that he really expects too much from his American relatives in the way of "understanding life". In other words, he demands that they make material sacrifice for the sake of improving positional parameters, i.e. improving relations with their immigrant relatives which may or may not help them in an indeterminate future. Here, help could have come in the form of warm feelings that cannot be bought at any price.

It seems that these American relatives of his decided that this kind of positional sacrifice is not for them. Money can buy them almost "anything" at their old age (medical care, a good retirement home, etc). Potential help and warmth from their immigrant relatives in the future was rather uncertain and far removed. Moreover, it takes a rather sophisticated individual to realize a positional sacrifice in personal relationships. Otherwise, an individual may suffer not only from being unable to realize a better position achieved as a result of a positional sacrifice but also, as we know from experience, from the deterioration of his relationship with those to whom he had extended his help. The American relatives were not sophisticated at all. Yet they were smart enough to understand their inability to realize the advantages of a positional sacrifice. They were like a clever intermediate level chess player who understands the inexpediency of committing to a positional sacrifice in his own games.

§ 3. Logical and historical development of styles of play.

Proceeding from the general to the particular, the logical sequence in the evolution of styles of play can be represented in the following way. At the start of the game, we set up a strategic goal-directed protractive positional problem of strategic variety which is followed by a tactical one belonging to the same class. Initially, this leads us to a com-positional problem and then to a combinational one. We then move to a non-goal directed search problem. After that come the reactive methods of play.

The history of chess and the process of learning it follow the same above sequence except in reverse.

Beginners, for the most part, play a combinational style of chess. They learn about different recommendations on how to play the game (see aggregate recommendations - heuristics). After that they face a very difficult question: "What do I do next?" They want to avoid inconsistencies inherent in these recommendations and seek ways to make their play more coherent. This leads them to think that material superiority is a precondition for victory. In fact, some players recommend that a direct attack against enemy king be initiated right in the middle of the game.[108]

Having mastered the combinational style of play a player can begin to assimilate the positional style.[109] This approach makes sense for without the ability to convert positional advantage into a combination it makes no sense to strive for positional superiority especially when it requires a material sacrifice.

There is a certain parallel between the process of learning and the development of the methods of play in general. Reactive methods based on personal wisdom prevailed in the past. This pertains not only to isolated rules but also to the entire games played by the great masters. Next came the combinational style which dominated the game until the end of the nineteenth century. The play aimed at gaining a large material advantage through beautiful combinations improvised by the players. Achieving this goal virtually ensured that the opponent's position would crumble very quickly. Usually the game did not last very long - about 20-25 moves. It seems that A. Andersen (1818-1879) represents the peak of combinational style.

Another great player P. Morphy (1837-1884) was first to introduce an innovative positional approach which violated the then prevalent style of play used even by the greatest masters of the day.

Let us note that before Morphy the games of chess masters contained an intermediate stage between opening and combination which aimed at loosening up the position. A sacrifice or some other sharp move produced tension on the board, and although the future course of the game remained unclear, a search for combination was thus made possible.

According to Jacob Neishadt[110],

> "The masters of combinational school, (this is partly true of Anderson in the 1850's) in opening up the game, often did not know how the play would continue. They were

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[108] Capablanca, H-R, Chess Manual, Moscow: Fizcultura i sport, 1975, p. 36.

[109] "One should master the combination style before attempting to play positionally." Reti, R., A Modern Manual on Chess, Moscow: Fizcultura i sport, 1981, p. 6.

[110] Neishadt, J. Uncrowned Champions. Moscow: Fizkultura I Sport, 1975. p. 286

convinced that in a great majority of complicated positions - those in which their pieces were not obstructed by "damned" pawns - they would find a profitable combination. Morphy was the one who understood the error of this approach, and would open up lines only when he had an advantage in development."

In particular, Morphy went after small positional advantages; among other things, he was willing to trade heavy pieces so important in carrying out combinations for the sake of gaining a small positional advantage. Morphy was very successful for he was able to convert his positional advantage into a combination. According to a great chess player R. Reti Morphy became the most famous "of all the people who ever played the game".[111]

V. Steinitz (1836-1900) was able to conceptualize positional style of play thus making it accessible to other players. E. Lasker writes

"How novel, how surprising, how opposed to every sentiment of his time the conceptions of Steinitz must have been...

... The main principle in Stenitz's conception could be expressed in the following way: a master's plan should always be based on his evaluation of the position.

The style with which Steinitz began his career was strange and narrow, but it was the style predominating in his time... The play was wholly dominated by the feverish desire to make a rush against the hostile king and to this end furiously to assail the obstructions, regardless of the sacrifices required.

Surely, Steinitz's heart beat when for the first time the thought came to him that the master should not look for winning combinations, unless he believed, unless he could prove to himself that he held an advantage. That meant making no attempt at winning in the beginning of the game. And since Steinitz lived in a milieu where to play to win right from the start was considered the only honorable course to take, this thought must at first have had a timid reception in his mind and a hard time establishing itself."[112]

Besides developing the techniques of positional play E. Lasker introduced philosophy and aesthetics into the game. Subsequently positional style was perfected by R. Capablanca, A. Nimzovitch, and other chess greats. At the present time, positional style is the leading and quite diverse style of play. It includes non-goal as well as different types of goal directed positional problems .

As E. Lasker point out opposing different methods of play is characteristics only of beginners. In masters' play the two styles complement each other. The linear picture of the game's history presented above is really more complicated. At different stages of the game one style of play is combined with another which by itself may belong to an earlier period of the game's evolution. This coalition often represents a very powerful combination.

Things will become more clear if we view the process and the structural approaches to the classification of various stages together.

Middle game is the most important example in this connection. It is at this stage that we face some of the most perplexing problems whose solution has a general methodological significance.

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[111]Ibid., p.16.

[112]Lasker, E., Lasker's Chess Manual, Moscow: OGIZ, Fizkultura i Turizm, 1937, pp. 193-195 and Lasker, E., Lasker's Manual of Chess, Philadelphia: Mc Kay Co., 1947, p.197.

Designing procedures for formulating local problems presents the most difficult task. Indeed, a large number of rather mobile pieces and an astronomical number of possible variations in the middle game raises two issues: on the one hand, unlike the end game, there is no way to reach the final goal, i.e. to formulate a terminal position and an optimization procedure to reach it. On the other hand, the play cannot be reduced to a reactive method based on prior experience (as is the case in the opening).

Looking at the chess game from a global perspective we see that each stage of the game embodies different methods of play including the reactive method. It is interesting to note that the reactive approach characteristic of the early history of the game may come into play at the game's more mature phase. But here these techniques would be much more sophisticated being applied in combination with search procedures.

Church, R. and Church, K. note in this respect:

> "The most reliable method for choosing a move is based upon complete pattern recognition. Particularly in the openings, a chess player has associated many positions with specific moves. When the player recognizes the position as one which has been previously encountered, the associated move can be executed without any analysis. Such moves can be made quickly and accurately.
>
> In the middlegame, it is not likely that the player will have encountered the complete position previously, but aspects of the position may include familiar patterns. For example, certain tactical elements (e.g., pin, discovery, x-ray, skewer, overworked piece, trap, and fork), certain structural features of the pawn formation, and other features of the position may be immediately recognized as patterns. These patterns will not be associated with specific moves, but with goals that the player might attempt to realize, and various plans to achieve these goals.
>
> The recognition of winning and drawing endgame patterns may be an essential factor in the choice of moves at any stage of the game. A player who attempts to work out a complicated rook and pawn endgame that his opponent recognized would be at a substantial disadvantage. As in the case of middlegame features, endgame patterns will normally be associated with goals and plans rather than specific moves. The treatment of the legal move as the elementary unit of chess thinking is concrete but undoubtedly misguided."[113]

M. Botvinnik stressed the idea of using different methods of play at different structural stages of the game. Here is what he writes:

> "When a chess master plays a game he uses historical experience (his own and the experience of others) in four different ways:
>
> 1. In parrot fashion. This is characteristic of the opening. The theory of the opening is not subject to judgment in any of its parts, and the master makes his moves in the opening without inserting himself into the process, i.e., he moves as a parrot talks.
>
> 2. By the handbook method. In playing a game, the master seeks out, in his store of accumulated knowledge, the exact position occurring in the search. This library position has a score; since the positions coincide, the score of the variation is known, and the variation itself may be cut off. This usage of historical experience is characteristic of the endgame, but may be successful in the opening as well, when moves are transposed.

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[113]Church, R. & Church, K., "Plans, Goals, and Search Strategies for the Selection of a Move in Chess", Chess Skill in Man and Machine, ed. by P. Frey. New York: Springer-Verlag, 1977, pp. 136-137.

> 3. By the outreach method. This is based on an attempt to reach positions that the handbook method can use. The master looks up library positions having a favorable outcome and resembling[114] the search position. (See the Glossary of Terms for a definition of nearness.) Having found such positions, the master constructs his search so as to get them into the search tree if possible. Then he relies on method 2. Therefore this method is also characteristic of the endgame and, perhaps, of the opening.
>
> 4. By the associative method. This is based on a partial congruence of the position in the search with positions in the library. The master seeks out a fragment of a library position, i.e., a small group of pieces whose action has led to success. If the search position contains the same group of pieces in the same constellation, he includes the group in his search, in order to see whether it will lead to success in the current case. If the fragment has often been successful in the past, it is apt to succeed again. This method determines a direction for the search and on the average it saves resources while the search is under construction. It would appear that the associative method presents the only way to apply historical experience to the middle game and to complex endgames."[115]

I leave it up to a professional chess player F. Reinfeld to sum up our discussion of the styles of play in chess. While I do not agree with him completely he managed to express these ideas better than I.

> "The style is the man himself," says Buffon, and nowhere is the famous phrase (Le style est l'homme meme) more appropriate than in chess. Many men, many styles; and what is chess style but the intangible expression of the will to win? The universe of the chess master is not without its grimmer aspects, for it is a world of dog-eat-dog. Beauty in chess (like virtue) is its own reward; it is only the incidental by-product of relentless struggle.
>
> In such an atmosphere, the quality of objective appreciation is not seen too frequently. Yet differences in style may produce queer paradoxes".[116]

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[114]Maximum allowable (D_{max}): the largest number of discrepancies between two positions equal in material that is permitted if the two positions are to be regarded as near each other;
between two positions equal in material (DAB) : the sum of the lengths of the trajectories leading from one (initial) position to the other (library) position." (p.146).

[115]Botvinnik, M., The Solution of Inexact Problems, Moscow: Soviet Radio, 1979.

[116]Reinfeld, F., Nimzovich the Hypermodern, Philadelphia, David McKay Co., 1948, p.23.

Conclusion

The game of chess models some very important phenomena taking place in the world around us. Chess is a model of competition based on equal starting conditions of all the participants with the outcome depending exclusively on their respective abilities to solve non-trivial problems. At the same time, many rules in chess are rooted in other moral statutes reflecting the social structure of society.

Despite its simplicity, the game yields to an experimental verification of a number of very important procedures aimed at achieving a certain result. The need to develop these procedures stems from the impossibility of linking the initial state with the end result by carrying out an exhaustive search of all possible variations and then designing an optimization algorithm to determine the best course of action.

One of the greatest achievements of chess theoreticians has been the development and synthesis of various techniques and methods to help guide our local behavior. Analysis of local problems has revealed a great variety among these methods. They can be viewed from different perspectives and within a hierarchical scheme. This enables us to combine different approaches to help solve local and global problems: problems limited to one move as well as problems encompassing an entire stage of the game (such as an opening, a middlegame or an endgame). As human experience has shown it is the synthesis of this manifold of local problems rather than its unification that is a crucial prerequisite for designing an effective algorithm of the game.

Of all the various methods of play, the two that are most noteworthy are the reactive and the search methods. I think that the greatest achievement of the game has been the discovery of search procedures aimed at creating good positions - the forerunners of combinations. No other field was able to develop the concept of integrating parameters in such a complete and precise fashion. Parameters which enable us to link one position with another in the absence of a procedure used to estimate completely and consistently the impact of a given position on the future flow of the game. Parameters of integration include both primary and conjugate parameters: material and its evaluation, positional parameters and their respective weights. Integrating parameters represent arguments of the weight function used to estimate the value of a position. A very special feature of this function is that it incorporates material as well as positional parameters as independent variables.

Numerous ways of modifying the structure of the weight function bring diversity in the kinds of search problems we have to deal with. Goal-directed problems are a very important category. These problems include a specific goal of reaching a certain position conceived beforehand. This class includes an important subset of combination-like objectives aimed at gaining material superiority with the program of achieving this goal known .

Despite all the techniques that have been developed by the players and the extent to which human experience has been formalized, subjective understanding still plays a tremendous role in the game. It manifests itself most vividly in positional style when a player has to judge the beauty of a position based on his experience and understanding. It is the uncertainty about the future progress of the game that makes positional judgments so subjective: indeed, the position in question is going to be played by a particular player and no one can tell ahead of time what the outcome might be. A sacrifice in chess puts the objective and the subjective aspects of the game in relief. It is also quite feasible that uncovering the least beautiful or even ugly spots in opponent's position is very important in goal formulation, especially that of capturing the material.

In spite of everything that has been accomplished in chess including the development of computer chess, the game still maintains its original vitality. It also yields to interesting experiments in solving non-trivial problems called by M. Botvinnik "inexact problems." Chess players have accumulated a lot of experience in the game. We were able, at least partially, to convert this experience and various techniques into formal procedures used in computer algorithms. Still, much of what has been accomplished in chess is still very far from being translated into a formal language. This applies equally well to the methods of solving similar types of problems arising in other fields. Some major puzzles in chess are still unresolved: formalization of the goal set up by within a local problem, the set of positional parameters and their weights, aggregation of different parameters, etc.

Particularly troublesome is the problem of formalizing the learning process in chess: this is still the prerogative of man. The problem consists not only in designing formal algorithms to implement the already existing methods of defining local problems but in teaching the machine to perfect these methods and to invent new ones. In other words, we are talking about creating an algorithm of the game capable of improving. I use the term "improving" intentionally. It encompasses both external and internal sources of learning and promotes the idea of a machine being able to create new methods on its own accord.

Until now, the emphasis in algorithmizing the game of chess has been on designing chess programs from the outside by a team well-qualified individuals (players, mathematicians, programmers). The task of designing a program to improve the game algorithm from the inside was often reduced to the creation of self-learning algorithms, i.e. a second-level algorithms aimed at perfecting the first-level algorithm based on the information accumulated from playing the game.

This information is received from many sources: games played by human beings, by humans and machines, and games played by a machine with other machines or with itself.

A number of difficulties in designing a self-learning algorithm came to the surface. The major ones were: 1) setting up a standard positional problem, i.e. elaborating new positional parameters and improving upon their weight assignment, 2) lack of sufficiently sophisticated methods for setting goal-oriented local problems, 3) "overabundance" and the contradictory nature of the heuristics incorporated into the reactive methods of play. Judging by the amount of literature on self-learning algorithms[117] the progress in this area has been rather slow.

We should note another problem which lies outside the scope of the game itself. This is the creation of algorithms whose purpose is to develop new algorithms of the game rather than just improve upon the old ones.

It seems that one of the major obstacles in creating such algorithms or in improving upon the structure of the already existing ones (rather than just modifying the values of constant coefficients) is the fact that the learning process is not limited exclusively to chess.

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[117]For example, Nitshe, T., "A Learning Chess Program", Advances in Computer Chess, ed. by M. R. B. Clarke, no.3, New York: Pergamon Press, 1982, pp. 113-120.

Experience gotten from the game itself may be insufficient: what is important is the creative potential of the creator of the algorithms, both self-learning ones and those aimed at improving other algorithms.

> I want to quote E. Lasker's thoughts on this subject: "Steinitz did not deduce his concept from just studying master games; finding the basis for it in human practice in general, he later tried to reconcile it with a purely chess experience. His venture was crowned with success."[118]

There is no reason to doubt that eventually the game of chess will be formalized to such an extent as to make an algorithm which finds an optimal strategy quite feasible. Even prior to developing such an optimization algorithm we could come up with a program which would be stronger than the world champion. Progress in computer chess could contribute enormously to the development of human thought in general for the range of problems similar to chess is very wide. And just as people ride horses in the age of airplanes and automobiles they will continue to enjoy the game of chess in the age of computers.

[118]Lasker, E., Lasker's Chess Manual, Moscow: OGIZ, Fizcultura i Turizm, 1937, p.248.